

# Appraisal of Queuing Performance in Chester Mega Petroleum Station Enugu

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## Abstract.

The operation manager of Mega Petroleum Station was finding it difficult in managing their queuing system and was not able to determine the best number of servers that can serve arriving customers at various demand periods which affects their queue performance. This study was conducted at Chester Mega Petroleum Station in Enugu, Nigeria with the aim of addressing the identified problem. The result from the Queue Evaluation Environment showed that 8 servers gave the best system utilization values of 73.2% which is expected to reduce the respective customers waiting times (Ws) by 84.7% for the case study establishments. The result further showed that with 8 servers for the service systems, there will be no more need of server increase at the respective average arrival rates of customers/minutes and also the expected probabilities of system idleness for the case studies were negligible at 8 server utilization. The Queue Evaluation Environment was later adopted in developing a Decision Support System for the referenced service facilities. The model was recommended for PMS refill only.

**Key Words:** Queuing, waiting line, arrival rate, service rate, waiting time, system utilization, Chester mega station, probability and decision support system

## 1. Introduction

A queue is a waiting line for service that forms either due to inability to meet up with arriving demands as a result of insufficient service capacity or due to stochastic nature of customer arrival and demand. A queue could deteriorate to congestion if effective queue management decisions are

not implemented. Queuing theory is the mathematical study of waiting lines [1]. The theory permits the derivation and calculation of several performance measures which includes the average waiting time in the queue or the system, the expected number waiting or receiving service, the probability of encountering the system empty, having an available server or having to wait a certain time to be served and most importantly the system utilization [2]. As a result of its applications in industries, technology, telecommunications networks, information technology and management sciences, it has been an interesting research area for many researchers active in the field.

In recent times, queuing theory and the diverse areas of its applications has grown tremendously. Takagi (1991) considered queuing phenomena with regard to its applications and performance evaluation in computer and communication systems [3]. Obamiro applied Queuing Model in Determining the Optimum number of Service Facility needed in Nigerian Hospitals. He however achieved this by determining some queuing parameters which enabled him to improve the performance of the system [4].

Chinwuko and Nwosu adopted the single line multi-server queuing existing model to analyze the queuing system of First Bank Nigeria PLC. In their work, they suggested the need to increase the number of servers in order to serve customers better in the case study organization [5].

Ohaneme et al, proposed the single line multi-server queuing system which they simulated using c-programming to be adopted at NNPC Mega petroleum station in Awka, Anambra State in order to avoid congestion and delay of customers [1].

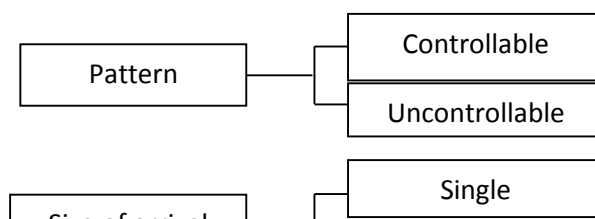
Presently, studies on performance evaluation of queuing have not been conducted at Chester Mega Petroleum Station Enugu. However, this work goes further in evaluating the performance of the queuing systems, creating a Queue Evaluation Environment that gives expected queue performance and developing a Decision Support System that recommends the best number of servers to use at various demand periods. Basic structure of queuing model can be separated into

input and output queuing system, which include queue that must obey a queuing rule and service mechanics [6].

**1.1. Input and Output Process:** Input process is known as the arrival process. Customers are known as arrivals which are generated one time by an input source randomly from finite or infinite population. These customers enter the queuing system and join a queue to be served. The required service is then performed for the customer by the service mechanism, after which the customer leaves the queuing system [6]. The provision of services using certain rule and discharge of customers is referred to as output process. Another fact worth mentioning here is that the key word in queuing models is “average”. It takes the average of the random numbers of customers arriving [7].

**1.2. Queuing System Characteristics:** The queuing system consists primarily of the waiting line(s) and the available number of servers. Factors to consider with waiting lines include the line length, number of lines, and queue discipline. Queuing phenomenon comprises of the following basic characteristics: Arrival characteristics; Queue or the physical line itself; Number of servers or service channels; Queue discipline; Service mechanism; Capacity of the system; and Departure [8].

**1.3. Waiting Line or Queue:** A waiting line or queue occurs when customers wait before being served because the service facility is temporarily engaged. A queue is characterized by the maximum permissible number of customers that it can contain. Queues are called infinite or finite. An infinite queue is one in which for all practical purposes, an unlimited number of customers can be held there while a finite queue refers to the limited-size customer pool that will use a service system and, at times, form a line [9].



**Figure 1: Arrival Characteristics in Queue**  
**Source: Adopted from Davis et al., [10]**

**1.4. Types of Queuing Systems:** There are four major types of queuing system. Lapin broadly categorized queuing system structures into the following [11].

- **Single-server, Single-phase system:**

This is a situation in which single queue of customers are to be served by a single service facility (server) one after the other. An example is bottles or cans of minerals or beer to be cocked in a production process.

- **Single-server, Multiple-phases System:**

In this situation, there's still a single queue but customers receive more than one kind of service before departing the queuing system. For example, in the university, students first arrive at the registration desk, get the registration done and then wait in a queue for their forms to be signed, after signing; they join another queue for submission.

- **Multiple-servers, Single-phase System:**

This is a queuing system characterized by a situation whereby there is a more than one service facility (servers) providing identical service but drawn on a single waiting line. An example is a petroleum service station.

- **Multiple servers, Multiple-phases System:**

This type of system has numerous queues and a complex network of multiple phases of services involved. This type of service is typically seen in a hospital setting, multi-specialty outpatient clinics, patient first form the queue for registration, and then he/she is triage for assessment, then for diagnostics, review, treatment, intervention or prescription and finally exits from the system or triage to different provider [7].

**1.5. Performance Measures of a Queuing System:** Hillier and Lieberman put forth the following performance parameters in a queuing system [6]:

- **System Utilization (P):** System Utilization is the most important measure of a queuing system. It is the ratio of system capacity used to available capacity. It measures the average time the system is busy. System utilization of zero means that there is nobody in the system. On the other hand, a system utilization of one or more signifies that there is infinite number of people on the waiting line. This means that the available servers cannot cope with the arriving demand. Thus something has to be done on the service facility. Egolum, further stated that the best value of system utilization should be greater than 0 but less than 0.8 i.e. 80% [12].
- **Mean Number in the system ( $L_s$ ):** Mean number in the system is the average number of system users (entities) in the system; it includes those in the queue and those being served by the server(s).
- **Mean Number in Queue ( $L_q$ ):** Mean number in the queue is the average or expected number of system users in the queue (waiting line), waiting for their turn to be served.
- **The average waiting time for an arrival not immediately served ( $W_a$ )**
- **Mean Time in System ( $W_s$ ):** Mean time in the system is the expected value or average waiting time an entity will spend in the queuing system. It includes the average time waiting for service to begin and the average service time.

- Mean Time in Queue ( $W_q$ ): Mean time in the queue is the expected value or average time an entity will spend in the queue, waiting for service to begin.
- probability of zero customers in the system ( $P_0$ )
- Probability of waiting ( $P_w$ ): This is the probability that an arrival will have to wait for its service to begin.

The aim of this study is to address the queuing problem at Chester Mega Petroleum Station by developing a Decision Support System that recommends the best number of servers needed to be engaged at various demand periods.

The structure of the studied system is shown in figure 2. The structures can be approximated as a single-line multi-server queuing systems. . At the Chester Mega Petroleum Station, there are six dispensers i.e. fuel metering pumps ( $S_1$  to  $S_6$ ) in the system. Each of the fuel dispensers has two nozzles. This means that at full capacity of operation the service facility should be considered a twelve-server system.

## **2. Methodology.**

There are two major techniques of research methods; they are qualitative research method and quantitative research method. The research method used in this work was the quantitative research approach. The single line multi-server queuing model was adopted for developing the results of the queue performance. This model was adopted because it showed a good representation of the model structure of both case studies of queuing systems.

### **2.1. Data Analysis**

The data generated was first organized and descriptive statistics was used to compute the total average arrival rates and total average combined service rates for the year. The service rates per server of both facilities were established and the single line multi server queuing model was coded in Microsoft Excel using 2 – 12 servers (i.e. when  $M = 2 - 12$  servers) in creating the Queue Evaluation Environment that generates the expected queue performance results at the respective average arrival rates of customers in the referenced service facilities. The results generated were validated using MATLAB (version 7.10.0.499: R2010a). The essence was to see if both results

corresponded with each other. The Queue Evaluation Environment was later adopted in developing the decision support system using the application of Microsoft Excel.

## 2.2. Models Applied for the Queuing Analysis

Based on the assumptions of the single line multi-server queuing model, the expressions for the performance measures which are derived from the analysis of the birth-and-death models, (Blanc, 2011 [13]; Sztrik, 2011 [14]; and Nain, 2004 [15]) are;

- i. The average utilization of the system:

When  $m = 6$  is

$$P = \frac{\bar{\lambda}}{\mu_c} \quad (1)$$

When  $m = 2 - 12$  is

$$P = \frac{\bar{\lambda}}{M(\bar{\mu})} \quad (2)$$

- ii. The probability that there are no customers in the system is

$$P_0 = \left[ \sum_{n=0}^{M-1} \frac{\left(\frac{\bar{\lambda}}{\mu}\right)^n}{n!} + \frac{\left(\frac{\bar{\lambda}}{\mu}\right)^M}{M! \left(1 - \frac{\bar{\lambda}}{M\mu}\right)} \right]^{-1} \quad (3)$$

- iii. The average number of customers waiting for service.

$$L_q = \frac{\bar{\lambda}\mu\left(\frac{\bar{\lambda}}{\mu}\right)^M}{(M-1)!(M\mu-\bar{\lambda})^2} P_0 \quad (4)$$

- iv. The average number of customers in the system.

$$L_s = L_q + (\bar{\lambda}/\bar{\mu}) \quad (5)$$

- v. The average time a customer spends in line waiting for service

$$W_q = \frac{L_q}{\bar{\lambda}} \quad (6)$$

- vi. The average time a customer spends in the system.

$$W_s = \frac{L_s}{\bar{\lambda}} \quad (7)$$

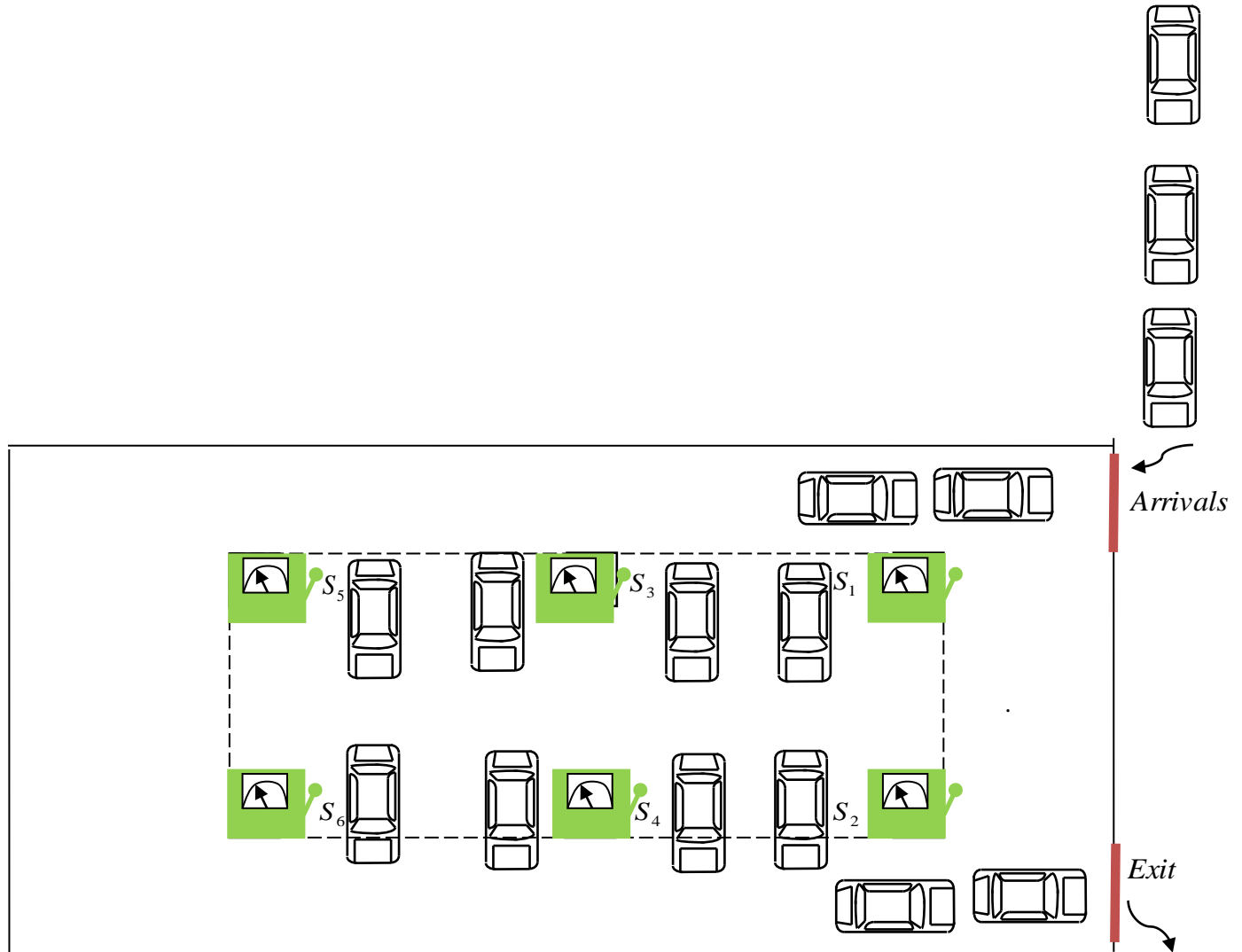
- vii. The average waiting time of a customer on arrival not immediately served.

$$W_a = \frac{1}{M\mu - \bar{\lambda}} \quad (8)$$

- viii. Probability that an arriving customer must wait

$$P_w = \frac{W_Q}{W_a} \tag{9}$$

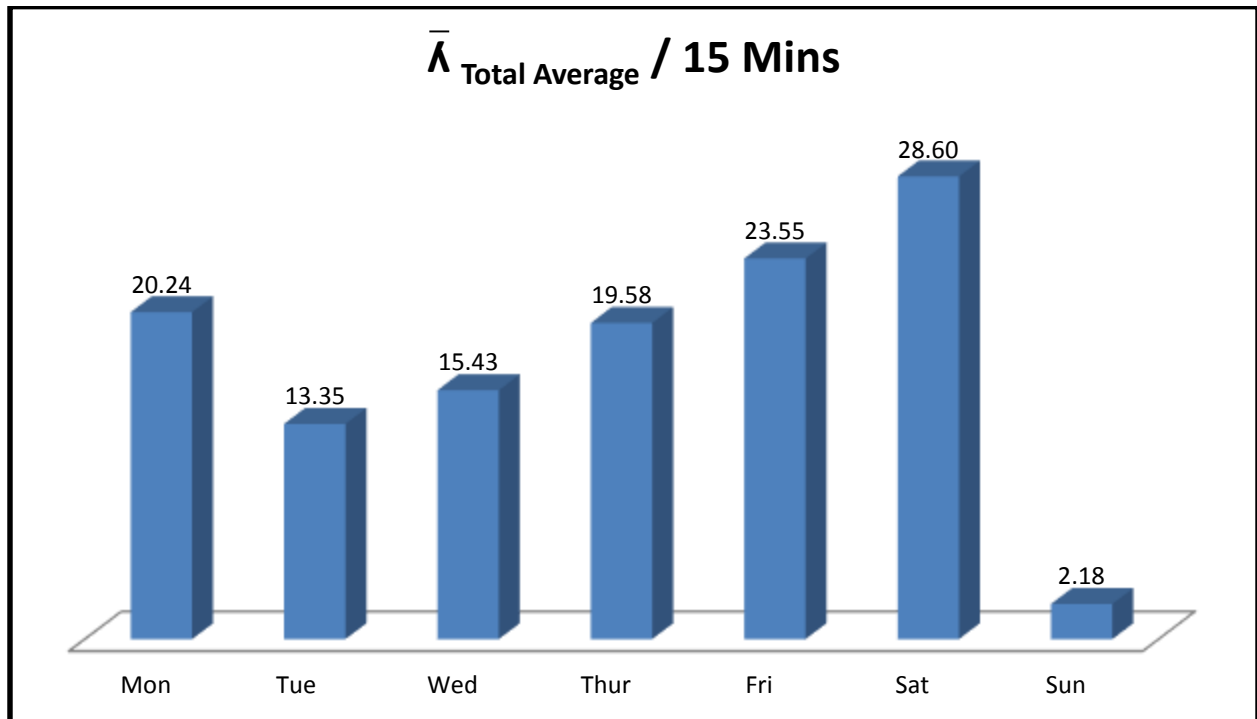
It is seen that these performance measures depend on two basic queue parameters, namely;  $\bar{\lambda}$  and  $\bar{\mu}$ . Given  $\bar{\lambda}$  and  $\bar{\mu}$ , the values computed for these measures gives an indication of how well the referenced service facilities handle the volume of arriving customers.



**Figure 2: Structure of the PMS Dispensary pump system of the studied Chester mega petroleum station Enugu.**



### 3. Data Analysis Result of Chester Mega Petroleum Station in Enugu



**Figure 3: Total Daily Average Arrival Rate / 15 Minutes**

In figure 3, the bar chart shows the total daily average arrival rate of customers per 15 minutes for the year from Monday to Sunday at Chester Mega Petroleum station Enugu. From the chart, it is observed that Saturday was with the highest arrivals which shows that the mega station is being patronage more by customers on Saturdays than the rest of the days being the fact that Saturday is a work free day for civil servants and most public servants so customers buy large quantity of PMS to last them for the week day activities as well as weekend travels. While Sunday was with the lowest arrivals which shows less patronage of customers being the fact that Sunday is a worship day for Christians and the Mega Station don't always open for service on that day and also customers most especially civil and public servants must have bought large quantity of PMS on Saturday and Friday to last them for the week. The week days (i.e. Mondays to Fridays) were mostly patronage more by commercial transporters

**Table 1: Weekly Mean Arrival Rate of Customers, Weekly Mean Combined Service Rate of Customers and Mean Number of Servers Engaged at Chester Mega Petroleum Station Enugu for the year**

		Weekly Average Arrival Rate Per Mins	Weekly Average Combined Service Rate Per Mins	Weekly Average Number of Servers Being Used (M)
DEC	1st Week	0.878	0.9095	7
	Last Week	0.3625	0.4071	3
JAN	1st Week	0.519	0.5911	5
	Last Week	1.2863	1.3042	5
FEB	1st Week	1.2744	1.297	8
	Last Week	1.2542	1.2887	6
MAR	1st Week	1.1798	1.206	6
	Last Week	1.225	1.2488	6
APR	1st Week	0.9137	0.9333	7
	Last Week	1.1982	1.2214	7
MAY	1st Week	1.2369	1.2577	8
	Last Week	1.256	1.2821	7
JUN	1st Week	1.2238	1.2554	6
	Last Week	1.2488	1.2762	6
JUL	1st Week	1.2601	1.3	8
	Last Week	1.1	1.1226	7
AUG	1st Week	1.3458	1.3756	6
	Last Week	1.3137	1.3345	6
SEP	1st Week	1.3506	1.3792	7
	Last Week	1.2792	1.3089	7
OCT	1st Week	1.3137	1.3411	6
	Last Week	1.3054	1.3286	6
NOV	1st Week	1.3976	1.4155	7
	Last Week	1.3762	1.394	7
Total		28.0989	28.7785	154
Average		1.1708	1.1991	6

Total average arrival rate for the year =  $\frac{\sum \bar{\lambda}(\text{Weekly})}{24} = 1.1708$  cars/minutes

Total average number of servers being used for the year =  $\frac{\sum \bar{M}(\text{Weekly})}{24} = 6$

Total average combined service rate for the year =  $\frac{\sum \bar{\mu}_c(\text{Weekly})}{24} = 1.1991$  cars/minutes.

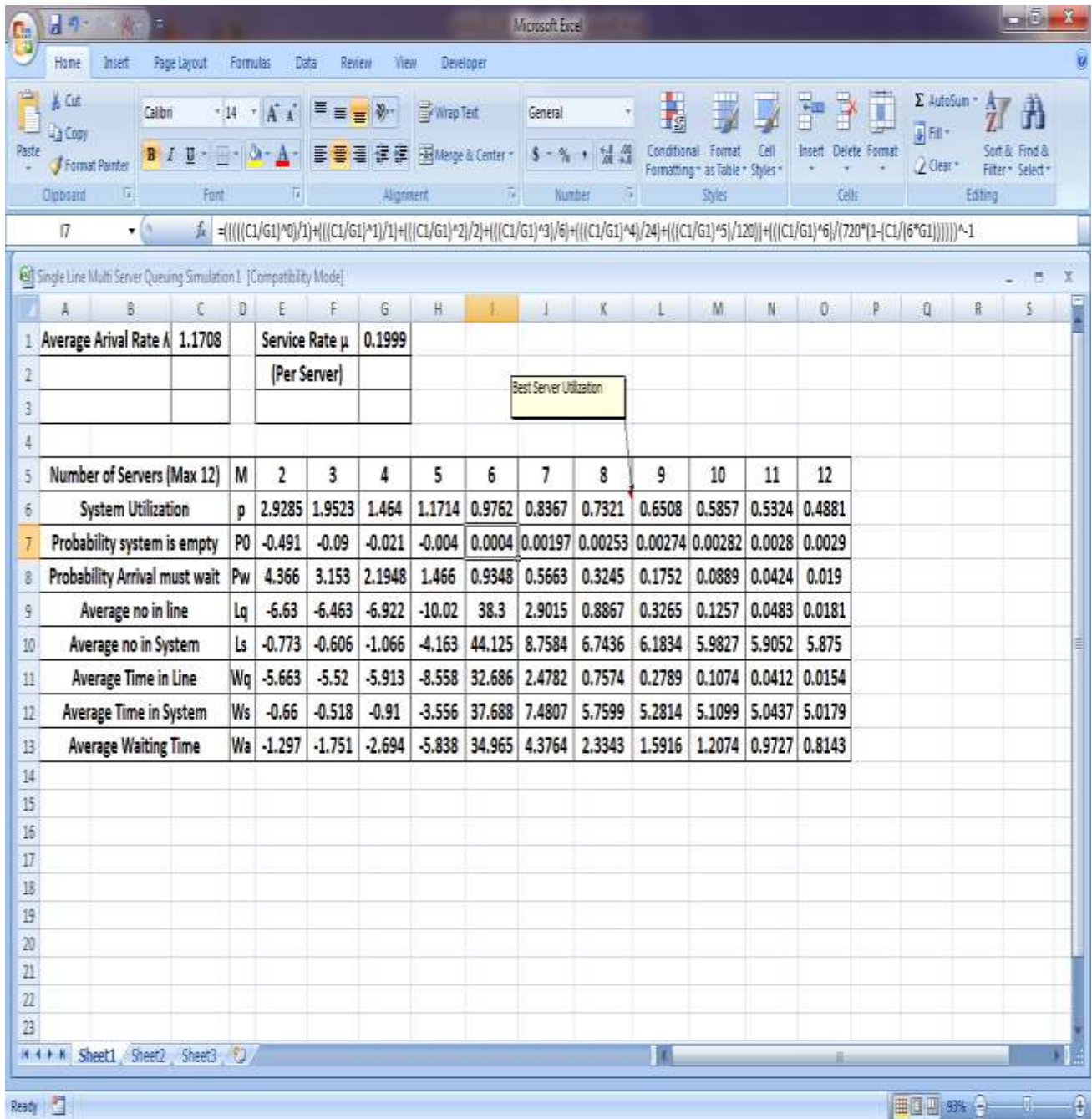
It is assumed that each server contributes an average service rate of  $\frac{\bar{\mu}_{c(Year)}}{\bar{M}}$  cars/minutes. Where  $\bar{M} = 6$ , and  $\bar{\mu}_{c(Year)} = 1.1991$  cars/minutes. This implies that each server contributes an average service rate of 0.1999 cars/minutes in the service facility

#### 4. Presentation of Queue Performance Evaluation Results

The results of the performance measures of the queuing system are presented below.

**Table 2: Results of the performance evaluation of the queuing system with parameters  $\bar{\lambda} = 1.1708$  cars/minutes and  $\bar{\mu}_c = 1.1991$  cars/minutes when  $(M= 6$**

Average Arival Rate $\lambda$	1.1708		Average Combined Service Rate $\mu_c$	1.1991
System Utilization	$\rho$	0.9762		
Probability system is empty	$P_0$	4E-04		
Probability Arrival must wait	$P_w$	0.9348		
Average no in line	$L_q$	38.3		
Average no in System	$L_s$	44.125		
Average Time in Line	$W_q$	32.686		
Average Time in System	$W_s$	37.688		
Average Waiting Time	$W_a$	34.965		



**Figure 4: Queue Evaluation Environment displaying the results of the queue performance when  $\bar{\lambda}$  is fixed,  $\bar{\mu}= 0.1999$  cars/minutes (per server) and  $M = 2 - 12$  servers**

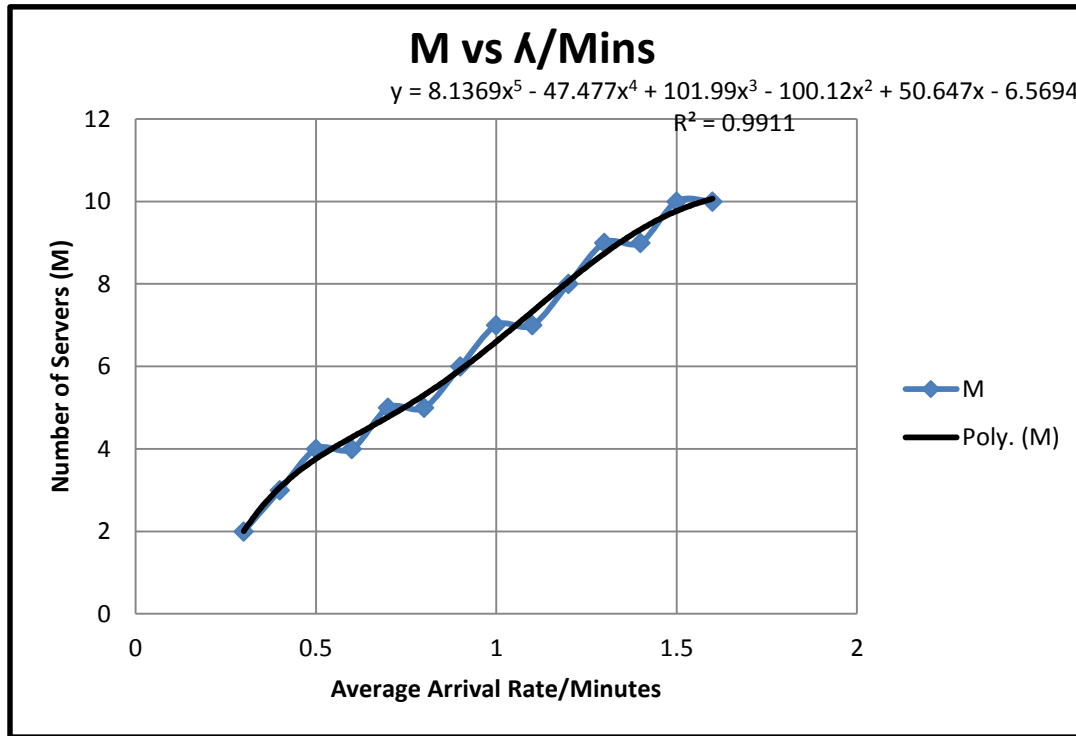
From figure 4, the charts of the queue output results were developed using the application of Microsoft Excel and trend line was used to test for the best goodness fit in developing the relationship that exists best between the queue output results.

### 5. Development of the Decision Support System for the Case Study

From the Queue Evaluation Environment created, the service rates (per server) of each of the referenced facilities were fixed and arrival rates were simulated using 2 – 12 servers to see the expected queue performance and to determine the best number of servers that gives the best system utilization value at various arrival rates of customers. The summary result outputs were plotted on a chart using the application of Microsoft Excel and trend line was used to test for the best goodness fit between the dependent variable i.e. Number of Servers (M) and the independent variable i.e. Average Arrival Rates/Minutes ( $\lambda$ ). See summary result output and charts below.

**Table 3: Summary result output of simulated arrival rates of customers/minutes (Chester Mega Petroleum Station Enugu)**

$\bar{\lambda}/Mins$	Best Server Utilization
0.3	2
0.4	3
0.5	4
0.6	4
0.7	5
0.8	5
0.9	6
1.0	7
1.1	7
1.2	8
1.3	9
1.4	9
1.5	10
1.6	10



**Figure 5: Scatter Plot of Number of Servers (M) vs. Average Arrival Rate/Minutes**

From figure 5, the scatter plot shows the number of servers plotted against average arrival rate/minutes in Chester Mega Petroleum Station Enugu. From the chart, it is observed that the number of servers is also expected to be stepping up as average arrival rate increases and the best fit between the two variables i.e. Number of Servers (M) and Average Arrival Rate ( $\lambda$ ) is a nonlinear polynomial function in fifth order as depicted in the chart.

## 6. Discussion of Results

From the analysis, table 2, shows the results for the performance measures of the queuing system as seen at the Chester mega petroleum station Enugu. From the results, it was also discovered that with an average number of 6 servers with average combined service rate ( $\bar{\mu}_c$ ) of 1.1991 cars/minutes and average customer arrival rate ( $\bar{\lambda}$ ) of 1.1708 gave a system utilization (P) of 0.9762 which gives a percentage system utilization of 97.62%, while the probability of the system being empty and the probability of waiting gave 0.0004 and 0.9348 respectively, this means that when service commences, the system is never idle and a customer must wait before receiving service with a 93.48% probability. However, the average number of customers in line and the

average number of customers in system including any being served gave 38.3 and 44.125 respectively. Furthermore, the average waiting time of customers in line, the average waiting time of customers in the system including service and the average waiting time of a customer on arrival not immediately served gave 32.686, 37.688 and 34.965 minutes respectively.

The results of table 2, for the case study showed that the system was heavily utilized at an average of 6 servers because system utilization was almost 100%. This resulted to the longer waiting time of customers experienced at both service facilities. However in respect of this, the service rate per server were determined for the case study and a Queue Evaluation Environment was created using 2 – 12 servers to see the expected queue performance and to determine the best number of servers that gives a good trade-off between system utilization and waiting time at the collected average arrival rates of customers in the referenced service facilities.

The results from the Queue Evaluation Environment showed that 8 servers gave the best system utilization values of 0.7321 which is expected to reduce the respective customers waiting times ( $W_s$ ) by 84.72% for the case study establishment. This is based on the statement of Egolum, which says that system utilization should be greater than 0 but less than 0.8 [12]. From the charts of system utilization versus waiting time plotted for the case study, it is observed that there's no significant decrease in waiting time anymore from system utilization value of 0.8, which shows that waiting time has reached its optimum at the respective best server utilization values of 0.7321 for the referenced service facility. This shows that there will be no need of making use of more than 8 servers at the respective average arrival rates of customers in the referenced service facility. Also the expected probability of system idleness is negligible at 8 server utilization because at that point, probability of system idleness has also reached its optimum and it no longer has any effect on the service systems.

From the study, it was also revealed that system utilization drops as number of server's increases; the probability of system being empty increased to optimum as number of server's increases; the probability of an arrival waiting reduces as number of server's increases; the average number in line and average number in system drops to optimum as number of server's increased; the average time in line, average time in system and average waiting time drops to optimum as number of server's increased.

Finally, from figure 5, the model for the decision support system was developed using trend line analysis. For Chester Mega Petroleum Station Enugu is given by:

$$M = 8.136\lambda^5 - 47.47\lambda^4 + 101.9\lambda^3 - 100.1\lambda^2 + 50.64\lambda - 6.569(4.34)$$

## 7. Conclusion

The evaluation of queuing system in an establishment is very essential for the betterment of the establishment. Most establishments are not aware on the significance of evaluating their queue performance. The implication of this is that operations managers are not able to determine the best number of servers to engage for service at various demand periods which affects their queue performance. As it concerns the case study establishment, the evaluation of their queuing system showed that their service system was over utilized which resulted to customers spending longer time than necessary before receiving service. However, the need of creating a Queue Evaluation Environment to find out the number of servers that gives the best server utilization at the collected average arrival rates became very essential. From the Queue Evaluation Environment, using 8 servers at the collected average arrival rates of customers in the referenced service facilities gave a good trade-off between system utilization and waiting time which is expected to reduce the waiting time of customers in the system while server idleness is neglected. In conclusion, the Queue Evaluation Environment created and the decision support system developed for the case study establishment will go a very long way in addressing their queuing problems.

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