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Controllability and Observability Criteria for Systems Described by Fractional Differential Equations

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Abstract: Based on the state response of fractional order singular linear systems with impulses, the sufficient and necessary conditions for complete controllability and observability of fast subsystems are studied and given, and the criteria for complete controllability and observability of fast subsystems are further established. These assumptions are too strong to synthesize the controllability of slow subsystems and fast subsystems. The method proposed in this paper does not need these assumptions, The approximate controllability of Hilfer fractional order integro differential equations is studied by using the order method. The controllability and observability criteria of the system described by fractional order differential equations are derived. When the rank of its controllability discrimination matrix M and observability discrimination matrix N is full, the fractional order system is controllable and observable.

Keywords: Controllability; observability criteria; fractional differential equations

1. INTRODUCTION

The essence of fractional derivative is weak singular integral of variable or its integer derivative. The kernel function in the definition of fractional derivative is called memory kernel function, which reflects the memory characteristics of fractional order system. The fractional order system is suitable for describing a variety of physical processes with the characteristics of "process memory" and "historical heredity"

Zhou et al. gave the definition of mild solutions to Riemman Liouville fractional differential equations and discussed the existence of mild solutions using the theory of noncompact measures. Hilfer extended the Riemman Liouville fractional derivative and proposed Hilfer fractional derivative. The derivative includes Caputo fractional derivative, The Riemman Liouville fractional derivative is also included. Then, aGu et al. M studied the existence of mild solutions of Hilfer fractional differential equations.

More work on the existence of mild solutions of fractional differential equations. Fractional calculus is a theory about differential and integral of any order. It is unified with integral calculus and is a generalization of integral calculus α D α T to mark differential and integral operators of non-integer order, where α It is an arbitrary real number. It is very important to study advanced differential equations in industries, bioengineering, economics and other related fields. Because advanced variables and integrals are ubiquitous, but it is difficult to deal with if we copy the construction of the comparison principle in G.T.Wang and [66]. This urges us to construct a new comparison principle, based on this new comparison principle, the existence of extreme solutions is proved by monotone iteration technique and upper and lower solutions.

Computed values of the output sequence is shown below.

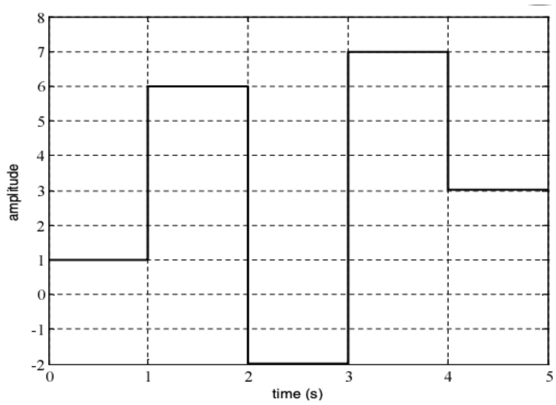

Figure. 1 Computed values of the output sequence. (Figure from Internet)

2. THE PROPOSED METHODOLOGY

2.1 Fractional Order Singular Linear Systems and Their Controllability (Observability) Concept

In this paper, we prove the necessary and sufficient conditions for the complete controllability and observability of fractional order singular linear systems with impulses and give the rank criteria for judging the complete controllability and observability of fractional order singular linear systems with impulses, which provide the basis and basis for the research and application of fractional order singular linear systems. The purpose of this paper is to study the approximation controllability of system (1.1).

It is worth noting that in the literature, the author discusses the approximation controllability of fractional order differential equations under the assumption that the nonlinear term is uniformly bounded, and the corresponding fractional order linear system is approximation controllability

Different from the above methods, the method proposed in this paper does not need these assumptions. It uses methods similar to those in the literature and makes necessary modifications to make it applicable to systems. Systems that can accurately describe their dynamic performance with fractional order differential equations are called fractional order systems. Integer order systems are special cases of fractional order systems. The mathematical models that describe fractional order systems include fractional order differential equations Fractional order transfer function and fractional order state space expression, etc.

Let the system be described by the following fractional order differential equation with sequential differentiation and initial value: because the structure presented by this equation set has a profound physical background and the real mathematical model is extremely consistent with natural phenomena, and there are a large number of models in applied mathematics and engineering mathematics that can be attributed to the existence of solutions to the boundary value problem of the equation set, Therefore, it is of great value to study the existence of solutions to boundary value problems of fractional differential equations

2.2 Approximate Controllability

To investigate the complete observability of the system, the control input u (t) of the system is generally not considered, but the output y (t) of the system needs to be considered. At this time, the concept of observability in the form of fractional order generalized linear time invariant systems, such as (5), is very important, because in practical applications, state feedback control often encounters such difficulties that the state cannot be directly measured.

At this time, it is required to estimate the unavailable state variables to realize state feedback control. According to the research on observability of integer order linear systems, fractional order can not only solve the problems contained in integer order, At the same time, it has its own unique properties. This paper is based on the controllability of fractional order linear neutral time-varying systems and fractional order nonlinear systems.

However, in the literature, the author has not given how the system changes when the system contains multiple delays. The uniqueness of this paper is that the author studies the controllability and observability of time-varying systems with multiple delays. In this paper, the reader can clearly obtain the controllability conditions when the system is a single delay system. Obviously, there must be an input u (t), so that equation (10) holds for any x~20, if and only if the matrix QC2=[B2, NB2,..., NBh-12] is row full rank (at this time, the rank of the augmented matrix is equal to the rank of the coefficient matrix), that is, rankQC2=rank [B2, NB2,..., Nh-1B2]=n2.

3. CONCLUSION

For fractional order singular linear systems with impulses, this paper proves the complete controllability and observability theorems of fractional order singular linear systems with impulses by using the restricted equivalent transformation and combining the distributed solution of fractional order singular linear systems with impulses and gives a simple and practical rank criterion for controllability and observability of fractional order singular linear systems with impulses. Fractional order systems are observable. The conclusions obtained are useful for the analysis and synthesis of fractional order linear control systems. This paper only deals with fractional order linear time invariant control systems. The controllability and observability of fractional order linear time varying control systems are worthy of further study.

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