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A Comparative Study of Heuristic Algorithms to Solve Flow Shop Scheduling Problem with Fuzzy Processing Time

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Abstract: Scheduling is the process of allocating resources over time to competitive activities such that a certain number of goals can be achieved while the given constraints can be satisfied. This paper aims to compare and analyze the performance of two heuristic algorithms in solving flow shop scheduling problem with fuzzy processing time to minimize the makespan and the mean flow time. The heuristic algorithms used are namely Palmer's and Campbell, Dudek, and Smith (CDS) algorithms. The processing times of all the jobs on machines are represented by triangular fuzzy numbers. In order to evaluate the performance of the heuristic algorithms, 12 benchmark problems of small size are used and compared using a MATLAB program. The computational results show that the CDS algorithm slightly outperformed Palmer's algorithm.

Keywords: Flow shop scheduling problem, Fuzzy processing time, Heuristic, Make span, Mean flow time.

1. INTRODUCTION

Scheduling plays an important role in the planning and operation of a manufacturing and production systems. It is a process of making decisions relating when, where, and how much workload can be divided among different resource requirements to achieve a certain number of goals [1]. The importance of the scheduling process has grown significantly due to the great development in production and manufacturing sectors. Thus, the research into scheduling problems has attracted the attention of many researchers in the last decades. The flow shop scheduling problem (FSSP) is one of the most widely studied scheduling problems, which can be defined and formulated using a variety of resources, tasks, and constraints [2] In the most studies concerned with the flow shop scheduling problems, processing times were taken as fixed and known value. But in the real-world application, information is often uncertain and imprecise. Although many researchers have succeeded in developing deterministic algorithms that have proven effective to obtain optimal / near optimal solution for flow shop scheduling problem [3[4][5][6][7]], these algorithms are often difficult to apply in many real problems, due to incomplete knowledge or uncertainty about the data such as processing times, set-up time, and due-date etc. Therefore, to overcome this the problem, fuzzy set theory can be used to deal with uncertainty inherent in data related to scheduling problems that are difficult to determine precisely. Recently, fuzzy set theory, introduced by Zadeh [8] in 1965, has been considered as an effective mathematical tool for modeling uncertainties in scheduling fields. Several studies have developed to solve flow shop scheduling problems with fuzzy processing times and fuzzy due dates. Ambika and Uthra [9] presented the branch and bound algorithm to obtain the job sequence with minimum makespan for three-machine flow shop problem with triangular fuzzy processing times. Another study [10] developed a new heuristic algorithm for minimizing the makespan for two-machine flow shops with triangular fuzzy processing time. The results compared and showed that the proposed algorithm performed the best performance in all cases. Jadhav et al [8] proposed an algorithm for solving flow-shop scheduling problem in fuzzy environment to optimize the total elapsed time. Job data are described by triangular and trapezoidal fuzzy number. The results have shown the efficiency and effectiveness of the proposed algorithm to find a job sequence with minimum makespan. Kurniawan and Farizal [4] developed a new method based on the concept of the CDS and NEH methods namely, NEHLPD, NEHLPD1, NEHLPD2 and then tested it on random cases. The results showed the NEHLPD outperformed other methods.

In a different study [2] applied matrix manipulation method in MATLAB to solve flow shop scheduling problem of n jobs on m machines under uncertain processing time. The problems have been considered for comparative analysis with Palmer's heuristic, CDS heuristic & NEH heuristic. The computational experiments showed that the proposed code and NEH heuristic outperforms over the other heuristics to find out the minimum makes span through an optimal sequence.

The above-mentioned literature review indicates the continuous attention shown by the researchers in solving flow shop scheduling problems with the uncertain processing time using various methods. Therefore, in this study, the flow shop scheduling problem with triangular fuzzy processing times is considered based on heuristic algorithms is introduced. The main aim is to evaluate the performance of heuristic algorithms to find the best sequence on jobs on machines with minimize the makespan.

2. PRELIMINARIES OF FUZZY SETS

In this section, the basic concepts and results of triangular fuzzy number and arithmetic operations are presented.

Definition 1[11]**:**

- Fuzzy number: A fuzzy number is a fuzzy set $\tilde{A}$ with a membership function defined by $\tilde{A}$ = {(x, μA(x)): x ∈ A, $\mu_A(x)$ ∈ [0, 1]}. In the pair (x, $\mu_A(x)$), the first element x belongs to the classical set A, the second element $\mu_A(x)$, belong to the interval [0, 1], called Membership function.

- A fuzzy set A˜ $\tilde{A}$ on R must possess at least the following three properties to qualify as a fuzzy number,
  I. $\tilde{A}$ must be a normal fuzzy set;
  II. $\tilde{A}$ must be closed interval for every $\alpha \in [0,1]$;
  III. The support of $\tilde{A}$, $\tilde{A}$ must be bounded.

Definition 2[9]: Triangular Fuzzy Number:

The triangular fuzzy number $\tilde{A}$ is denoted by the formula (a, b, c), where a, b, c are real numbers and its membership function is given by,

$$\mu_A(X) = \begin{cases} \frac{x-a}{b-a}, & for\ a \le x \le b \\ \frac{c-x}{c-b}, & for\ b \le x \le c \\ 0, & elsewhere \end{cases}$$

According to the above relation, the triangular membership functions of a fuzzy number $\tilde{A}$ are used to represent the fuzzy processing time of jobs on machines where the membership value reaches the highest point at 'b', while 'a' and 'c' denote the lower bound and upper bound of the processing time respectively. The values a, b and c are interpreted as pessimistic, moderate and optimistic values of the processing time as shown in Figure 1.

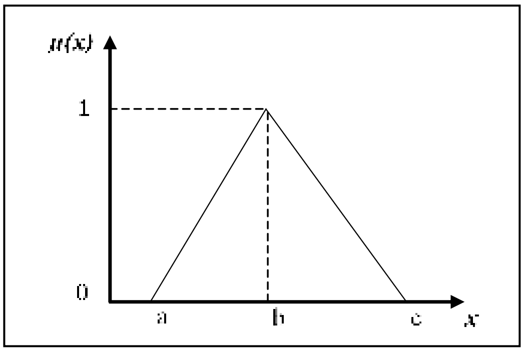

Figure1.Triangular Membership Function of fuzzy number.

Definition 3: Average High Ranking

Averge high ranking **(AHR)** is proposed by Yager's [8,9] which used to calculate the expected time of jobs on the machines. The formula of AHR can be calculated as follows:

$$h(A) = \frac{3b + c - a}{3}$$

Definition 4: Fuzzy Arithmetic Operations

Let $\tilde{A}$ = (a_1, a_2, a_3) and $\tilde{B}$ = (b_1, b_2, b_3) be two triangular fuzzy numbers. The basic arithmetic operations that can be performed on triangular fuzzy numbers are defined as follows [8,9]:

I. Addition: $\tilde{A} + \tilde{B}$ = (a_1, a_2, a_3) + (b_1, b_2, b_3) = ($a_1 + b_1$, $a_2 + b_2$, $a_3 + b_3$)
II. Subtraction: $\tilde{A} - \tilde{B}$ = (a_1, a_2, a_3) − (b_1, b_2, b_3) = ($a_1 - b_1$, $a_2 - b_2$, $a_3 - b_3$)
III. Multiplication: $\tilde{A} \times \tilde{B}$ = (Min ($a_1 b_1$, $a_1 b_3$, $a_3 b_1$, $a_3 b_3$), Max ($a_1 b_1$, $a_1 b_3$, $a_3 b_1$, $a_3 b_3$))

3. PROBLEM STATEMENT

Consider a set of n jobs to be processed in the same order on m machines. The processing times of all the jobs on machines are described by the triangular fuzzy numbers $\tilde{P}_{ij}$. Our aim is to find the sequence of jobs to be processed on machines using heuristic algorithms which include Palmer's and Campbell, Dudek and Smith (CDS) algorithm. The considered problem is based on the following assumptions:

1. All the jobs and machines are available at time zero.
2. Each job must be completed when started.
3. To make job on a second machine, it must be completed on the first machine.
4. Machines never break down and are available throughout the scheduling period.
5. Machines may be idle.
6. Setup times are known and are included in processing times.

The notations used throughout the paper are as follows:

n : Number of jobs.
m :Number of machines.
$\tilde{P}_{ij}$:Fuzzy Processing time of job i on machine j, i = 1, 2,...n and j = 1, 2, ..., m.
$\tilde{C}_{ij}$: The fuzzy completion time for the i^{th} job on the j^{th} machine.
a_{ij} : Fuzzy processing time of i^{th} job on machine mj, i = 1, 2,3,....,n; j=1,2,..,m
A_{ij} : Processing time of the i^{th} job on j^{th} machine; i = 1,2, 3….,n ; j= 1,2,…m
$h_i(A_j)$: AHR of processing time of i^{th} job on j^{th} machine; i = 1, .., n; j= 1,2,…m.
$\tilde{M}$: Fuzzy makespan.
$M\tilde{F}T$: Fuzzy mean flow time.
(+) : Fuzzy addition.
SI_j : Slope index.

In this study, fuzzy make-span ($\tilde{M}$) and fuzzy mean flow time ($M\tilde{F}T$) are used as performance measurers that are computed by using the following equations:

The fuzzy makespan is calculated as

$$\tilde{M} = \underset{i\epsilon j}{m\tilde{a}x}\ \tilde{C}_{ij} \quad (1)$$

The fuzzy completion time for the i^{th} job on the m^{th} machine, is calculated as follows:

$$\tilde{C}_{ij} = m\tilde{a}x\{\tilde{C}_{i-1,j}; \tilde{C}_{i,j-1}\}(+)\tilde{P}_{ij} \quad (2)$$

Assuming job i-1 precedes job i in the sequence.

The fuzzy mean flow time is calculated as

$$M\tilde{F}T = \frac{\overset{n}{\underset{i=1}{(+)}}\ \tilde{C}_{ij}}{n} \quad (3)$$

4. DESCRIPTION OF COMPARATIVE ALGORITHMS

In the section, the steps of heuristic algorithms used to solve the considered problem using fuzzy processing time to obtain the best sequence of jobs on machines are explained.

4.2 Fuzzy Palmer's algorithm

D. S. Palme×r proposed a solution to the general (n, m) n jobs and m machines problem by computing a slope index to give priorities to jobs to proceed from one machine to another and then sequencing the jobs in descending order of the slope index[12]. The formula for the slope index SI_j is shown below.

$$SIj = -\sum_{i=1}^{m}\{m-(2i-1)\}a_{ij} \quad (4)$$

The steps of fuzzy Palmer algorithm are as follows:

Step 1: Converting the fuzzy numbers into crisp numbers by computing average high ranking (A_{ij}) of the processing times for all the jobs on all machines.

Step 2: Compute the slope SI_j for each job.

Step 3: Arrange the jobs as per the decreasing order of the slope.

Step 4: Determine the value of the makespan and the mean flow time.

4.2 Fuzzy Campbell, Dudek, and Smith (FCDS) algorithm

The Campbell, Dudek and Smith (CDS) algorithm creates a series of m- 1 auxiliary n-job, 2-machines problems, where Johnson's algorithm is then applied to each of these auxiliary problems [13].

This heuristic is a generalization of Johnson's two machine algorithm and it generates a set of m − 1 auxiliary n-job , 2-machine problems from an original m machine problem, then each of the generated problems are solved using Johnson's algorithm as mentioned Soltysova1 et al in [14]. The series of m -1 auxiliary problems is generated using the following steps:

Step 1: calculate the pseudo –machine processing times for each l^{th} auxiliary problem, l= 1, 2,…..,m-1 as:

$$\beta_{i2}^{l} = (+)\breve{P}_{ij} \quad (5)$$

$$\beta_{i2}^{l} = (+)\breve{P}_{ij} \quad (6)$$

Step 2: average high ranking (A_{ij}) of the fuzzy processing times is computed before applying Johnson's algorithm to the two pseudo machines using β_{i1}^{l} and β_{i2}^{l} as the processing times to get an optimal sequence.

Step 3: Calculate the makespan of the l-th sequence found in

Step 4: Compare the makespan of the m-1 sequences. Select the minimum make-span.

5. COMPUTATIONAL RESULTS

The computational experiment is carried out in order to evaluate the performance of the two heuristic algorithms. The makespan and mean flow time is calculated for each problem. 12 benchmark problems of small size taken from the previous studies are used. The present algorithms are coded in MATLAB over Intel(R2014a) core (TM) i3 CPU @ 2.20 GHZ computer with 2GB RAM. The results obtained are reported in Tables 1-2 respectively.

Table 1. Comparative results of the fuzzy makespan.

Problem size	FCDS Algorithm	Fuzzy Palmer's Algorithm
3×3	38,45,57	39,46,57
3×4	48,57,71	50,60,74
3×5	68,78,93	70,80,95
4×3	23,36,46	24,39,49
4×4	111,114,137	111,114,137
4×5	47,58,77	47,58,77
5×3	49,57,73	52,61,74
5×4	112,124,135	114,126,137
5×5	69,78,87	71,80,89
6×3	102,111,119	107,115,124
6×4	109,121,132	118,127,137
6×5	153,168,181	153,168,181
Average	**95.22**	**97.17**

Table 2. Comparative results of the fuzzy mean flow time.

Problem size	FCDS Algorithm	Fuzzy Palmer's Algorithm
3×3	28.67, 35, 43.67	26.67, 2.33,41.67
3×4	40, 48, 60.33	38.33,45.67,59
3×5	61.33, 69.67, 82	62, 70.33,84.33
4×3	13.75, 23.25,31	17.75,28.75,36.50
4×4	83.50,83.75,104	83.50,83.75,104
4×5	34.50, 44, 57.75	33.25,43.25,56.50
5×3	35.40,42.80,53.60	30,36.40,47.20
5×4	73.40,82.60,90.80	80.80,89.80,97.80
5×5	51,58,65	53.20,60.20,67.20
6×3	67.33,73.67,79.67	66,72.5,78.67
6×4	80.67, 89.0, 97.33	78.,86.83 ,95.5
6×5	113,124.33,133.7	113.33,124.8,134.33
Average	**70.67**	**70.75**

Based on the above table, it can be seen that the results obtained using the two heuristic algorithms were satisfactory. From that result, the average of fuzzy make span of the FCDS algorithm was 95.22 while the fuzzy Palmer's algorithm was 97.14. Also, the average of fuzzy mean flow time for the FCDS algorithm was 70.67 while the fuzzy Palmer's algorithm was 70.75. Thus, the FCDS slightly outperformed the Palmer's algorithm in the most benchmark problems, regardless of the size of problem, which can also be observed in Figures 2-3 respectively.

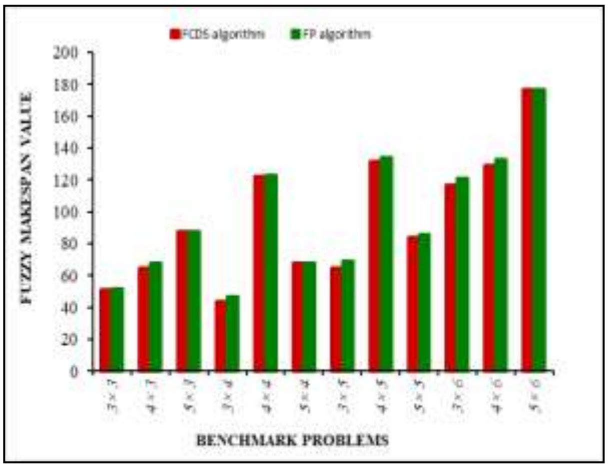

Figure 2. Comparative results of fuzzy makespan.

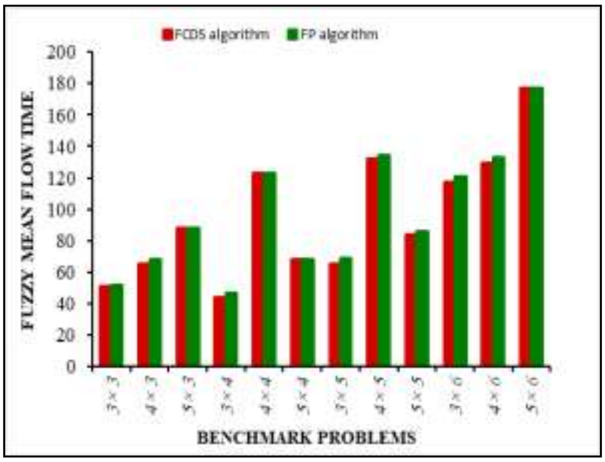

Figure 3. Comparative results of the fuzzy mean flow time.

5. CONCLUSIONS

This paper focused on application of heuristic algorithms for solving flow shop scheduling problem with fuzzy processing time. The heuristic algorithms used are namely Palmer's and Campbell, Dudek, and Smith (CDS) algorithms. The main aim is to compare and analyze the performance of two heuristic algorithms in solving FSSP to obtain the best sequence of jobs on machine that minimizes the performance measures considered. Twelve benchmark problems of small size taken from the previous studies are used and compared using a MATLAB program. The computational results show that the FCDS algorithm was better than the fuzzy Palmer's algorithm. In general, the two heuristic algorithms can be applied to different types of fuzzy numbers and in uncertain environments, also, can be compared with other heuristic algorithms to select the best solution.

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