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A Queuing Theory Approach for Monitoring Indoor Parking Management System

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Abstract: Smart indoor parking management systems represent an evolution of traditional parking setups. These systems provide real-time information to users, simplifying the search for available parking spots, while also giving operators the ability to monitor and control the system remotely. This research aims to apply queuing theory principles to model parking system dynamics. By implementing a queuing system model, arrival and service times can be predicted through detailed analysis and calculation. Additionally, a Graphical User Interface (GUI) is developed and integrated into the parking system, allowing operators to monitor parking lot status and view key statistics, such as arrival rates and service times. The GUI also enables parking reservations through manual control of parking inputs and outputs. Currently, most parking systems provide limited information, often requiring users to manually search for open spots. This research seeks to support parking operators in designing efficient systems that increase revenue while enabling convenient remote management, ultimately offering users a time-saving, stress-free parking experience.

Keywords: queue theory; poisson distribution; general distribution; arrival prediction; server utilization

1. INTRODUCTION

As Malaysia progresses into the era of globalization, vehicle ownership has become essential for most citizens. The number of vehicles in the country now exceeds the human population, with an annual increase of at least one million vehicles since 2021 [1]. This surge in vehicle sales has led to greater road congestion, necessitating a steady increase in parking spaces, both in open and indoor areas. However, limited space restricts the expansion of parking facilities, making it critical to maximize the use of each parking lot and control traffic flow systematically to avoid congestion in confined areas.

Numerous models have been developed to evaluate parking lot performance, analyze parking systems, and design solutions tailored to various needs. Queuing theory has proven especially effective in this context, as queuing models are both data-efficient and straightforward, allowing for quick and practical application. This simplicity and speed enable modelers to rapidly analyze and compare multiple service options, making queuing theory a preferred approach for optimizing parking systems. Consequently, many researchers have applied queuing theory related to parking management system in diverse ways, as illustrated in studies [2]-[8].

Li and Miao [2] introduced an automated stereo-garage parking system designed to reduce construction costs and improve overall performance. They implemented two distinct scheduling strategies: one focusing on maximizing garage efficiency and another aimed at minimizing customer wait times. By applying queuing theory, they simulated garage operations during peak usage periods to analyse service performance. Another study [3] presented an automated parking service utilizing vehicle communication with Road Side Units (RSU). This system applied an M/M/1 queuing model to calculate the total service time (TST) for the entire parking process.

Xie and Hlynka [4] developed a mathematical model to explore how the orientation of parked vehicles influences arrival and service times, providing valuable insights into the directional flow within parking lots. To forecast parking demand at individual parking lots, a traffic assignment model was proposed by [5], utilizing the M/M/s (∞) queuing model to estimate parking wait times. This model not only forecasts parking demand for each lot but also allows for the evaluation of the impact of various parking policies

Pandey and Hanchate [6] proposed an intelligent parking management system that integrates queuing theory and IoT technology. Using Python simulations, they analyzed system performance and worked to enhance the user experience. Abdeen et al. [7] applied an M/M/c/c queuing model to estimate the availability of parking spaces based on the arrival and service rates of vehicles, as well as the parking lot's capacity. In another study [8], researchers introduced a method to allocate parking spaces in school zones to reduce congestion during drop-off and pick-up times, utilizing a G/M/N queuing model. The study also proposed an optimization model to adjust both total and short-term parking capacities for better space utilization. Results indicated that the model effectively matches actual parking demand.

Based on research [2]-[8], queuing models are essential tools for optimizing parking system operations. These models help strike a balance between service capacity and associated costs—excess service capacity can lead to high operational costs, while insufficient capacity results in long wait times and related issues. By leveraging queuing theory, these models provide a framework to manage this balance effectively, offering valuable insights for designing and analyzing complex system challenges. Ultimately, queuing models contribute to the efficient management and optimization of parking systems.

In typical indoor parking zones, traffic is directed in a single lane to maximize space, causing drivers to slowly circulate while searching for available spots. This search process can create congestion, especially when multiple cars follow closely, and the frequent stop-and-go movement increases the risk of minor collisions. To address these issues, assigning each vehicle a designated parking spot upon entry directs drivers immediately to their spots, reducing search time and alleviating congestion. Building on previous work that applied queuing theory, this research proposes a smart indoor parking management system. This system provides users with real-time information on available spots, simplifying their search, and allows operators to monitor and control the system remotely.

This research aims to support parking operators in designing systems that not only maximize operational efficiency and boost revenue but also allow for seamless remote management. By implementing real-time monitoring and control capabilities, the proposed system empowers operators to efficiently oversee parking operations from a distance. This approach enhances user experience by providing a streamlined, time-saving process that reduces the stress associated with finding parking, ultimately delivering a smoother, more convenient parking experience.

2. OVERVIEW OF QUEUE THEORY

Queuing models offer mathematical and simulation-based methods for solving system performance issues, providing analysts with essential tools for designing and assessing queuing systems. When developing or optimizing these systems, analysts must consider trade-offs between server utilization and customer satisfaction, particularly in terms of wait times and queue lengths. Queuing theory and simulation enable predictions of system performance metrics, such as arrival rates, service demands, service rates, and server configurations.

As a branch of applied probability, queuing theory—also known as traffic theory, congestion theory, or the theory of stochastic service systems—focuses on the mathematical study of waiting lines. It allows for both macroscopic and microscopic modeling of processes involving queues, which typically consist of arrival, waiting, and service phases. A queuing system is often represented by the five-part code A/B/C/Y/Z, indicating the distribution of arrival times, service times, number of servers, system capacity, and queue discipline. Common symbols include G for general distribution, M for memoryless or exponential distribution, and D for deterministic times [9].

In a basic queue model, customers (inputs) arrive at a service facility according to a stochastic distribution, typically Poisson, and seek service. The facility may consist of one or more service channels arranged in series, parallel, or networked structures, with service times commonly following an exponential distribution. If a service channel is available, the customer is attended and exits upon completion; if not, the customer waits in line, following a set of rules such as balking, reneging, or queue discipline (e.g., priorities, jockeying). The primary challenge in queuing theory is probabilistic: given the rules for arrival, service, and queue behavior, analysts aim to determine the distribution of system states over time, including metrics like waiting times, idle intervals, and the fraction of delayed customers. A related statistical problem involves using observed system data to infer operational rules, particularly the input and service time distributions [10]

3. METHODOLOGY

3.1 Parking System Modelling

In this monitoring system for Parking Management Services, queuing theory principles are applied to manage the flow of vehicles. In a typical queuing model, customers arrive intermittently, join a queue, receive service, and then exit the system. Here, 'customers' refer to parking zone users, while the 'server' is the automatic ticket teller machine, which assigns parking spaces, issues tickets at entry, and retracts them at exit points. Queuing theory is used to analyze waiting line issues at the entrance, where the system must manage vehicles arriving randomly and requesting parking tickets. Mathematical models help determine relationships among car arrivals, waiting times at the ticket machine, ticket issuance, gate operations, and the time to exit the teller machine. Figure 1 shows the parking zone layout, featuring two entrance and two exit gates with single-direction traffic flow. The nearest parking lot is calculated based on proximity to the shopping mall entrance, located adjacent to the exit gates.

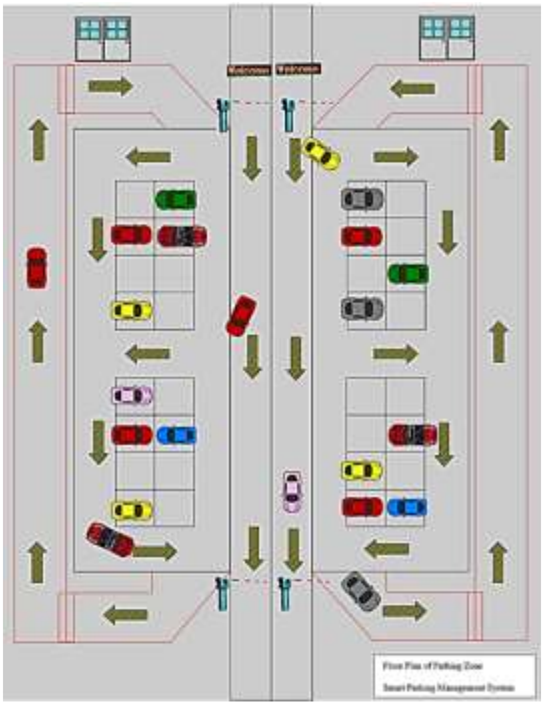

Figure1. Floor plan of the parking zone

In modeling this system, car arrival and service times at the entrance gate follow Poisson distributions, given the random flow of vehicles in the queue and the variable service time for each car. Departures are also assumed to be random, allowing vehicles to leave as they choose. The number of servers reflects the number of ticket machines, and the system's capacity is defined by the maximum queue length. This system's configuration, using Kendall's notation, is M/M/C/∞, with 'M' indicating a Markov or exponential distribution.

At the exit gates, both arrival and departure times are modeled using general random distributions, as vehicle arrivals at this point are unpredictable. With two servers at the exit and a queue capacity limited by the 64 available parking spaces, the system at the exit can be represented in Kendall's notation as G/G/2/64, where 'G' denotes general distributions, '2' is the number of servers, and '64' is the queue capacity.

The parking system can be represented by two subsystems, as shown in Figures 2 and 3: the entrance barrier gate and the exit gate of the parking area. The entrance subsystem employs an M/M/2 queuing model, where cars arrive from the external network at an exponential rate, with each car experiencing an exponential service time at the ticket machine. This subsystem has two barrier gates acting as servers, unlimited queue capacity, and an unrestricted queuing discipline. Cars from the

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main road join a queue for servers G1 and G2, as illustrated. Service operates on a First-In-First-Out (FIFO) basis, where the first car in line selects an available server. Once drivers obtain parking tickets, they proceed directly to their designated parking spots.

Before reaching the exit gate, cars merge into a single point for departure. The exit subsystem uses a G/G/2/64 queuing model, characterized by random arrivals from the parking area and random service times at the ticket machine. This subsystem includes two servers, represented by the exit barrier gates E1 and E2 on the first level. The queue capacity is capped at 64, aligning with the assumed 64 parking spaces depicted in Figure 1. Consequently, the total number of vehicles within the system cannot exceed the total parking capacity. Queuing discipline remains unrestricted, and when the first level reaches full capacity, cars are directed to the next level of the parking area.

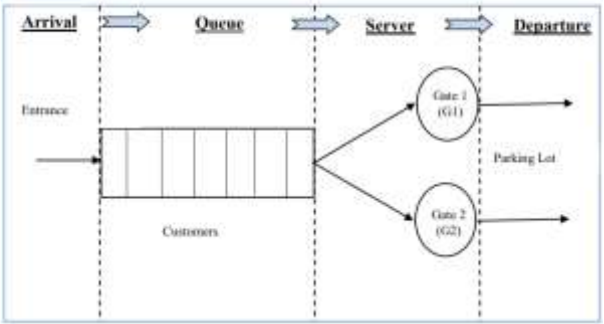

Figure 2. Queue model for parking entrance gate using M/M/2

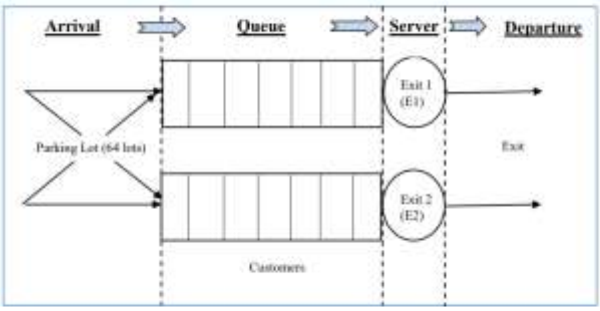

Figure 3. Queue model of parking exit gate using G/G/2/64.

The fundamental parameters required for calculating the indices of a queuing system's performance are the arrival rate (λ) and the service rate (μ). In practical applications, these parameters are typically determined using data gathered through statistical observation or evaluation [11],

- The arrival rate λ can be calculated as

$$\lambda = \frac{\lambda_{CH}}{3600} \quad (1)$$

Where λ_{CH} is the number of cars arrive in one hour

- For the case of random arrival, random service, C service channels and K maximum number of vehicles in the system, the probability of having zero vehicles in the system can be calculated as:

For $\frac{\rho}{c} \neq 1$

$$P_{0}= \left[\sum_{n=0}^{c-1} \left(\frac{1}{n!}\rho^n \right) + \left(\frac{\rho^c}{C!} \right) \left(\frac{1-\left(\frac{\rho}{c}\right)^{K-c+1}}{1-\frac{\rho}{c}} \right) \right]^{-1} \quad (2)$$

For $\frac{\rho}{c} = 1$

$$P_{0}= \left[\sum_{n=0}^{c-1} \left(\frac{1}{n!}\rho^n \right) + \left(\frac{\rho^c}{C!} \right) (K-c+1) \right]^{-1} \quad (3)$$

where server utilization, $\rho = \frac{\lambda}{\mu}$.

- The probability of having n vehicles in the system:

$$P_n = \frac{1}{n!}\rho^n P_o \qquad \text{for } 0 \le n \le c \quad (4)$$

$$P_n = \left(\frac{1}{C^{n-c}C!} \right) \rho^n P_0 \qquad \text{for } 0 \le n \le k \quad (5)$$

- Expected average queue length:

$$E(m) = \frac{P_0 \rho^c \left(\frac{\rho}{c}\right)}{c!\left(1-\frac{\rho}{c}\right)^2} \left[1 - \left(\frac{\rho}{c}\right)^t - \left(1-\left(\frac{\rho}{c}\right)\right) t \left(\frac{\rho}{c}\right)^{K-c} \right] \quad (6)$$

Where $t = K - c + 1$

- Expected average number in the systems:

$$E(n) = E(m) + c - P_0 \sum_{n=0}^{c-1} \frac{(c-n)\rho^n}{n!} \quad (7)$$

- Expected average total time:

$$E(v) = \frac{E(n)}{\lambda(1-P_K)} \quad (8)$$

- Expected average waiting time:

$$E(w) = E(v) - \frac{1}{\mu} \quad (9)$$

3.2 Software Development

For software development, National Instruments (NI) is utilized due to its simplicity in graphical programming, which allows for faster system development. The program is initiated when the user presses the ticket button on the automatic ticket dispenser before entering the parking area. The system then checks parking availability starting from the nearest lot to the furthest.

Dijkstra's Algorithm is implemented to determine the shortest path from the initial node to the destination node, passing through intermediate nodes. In this system, the initial nodes represent the parking lots, while the destination node is the mall entrance. First, all identified nodes are marked as unvisited. The system calculates the distance from the initial node to the nearest adjacent node and selects the node with the shortest distance. This selected node becomes the new initial node, and the process repeats until the destination node is reached and all nodes are visited.

The algorithm is then repeated to calculate the second shortest distance from each parking lot to the mall, continuing until distances for all parking lots are determined. This process identifies the nearest available parking lot for the user. The calculated distances are stored in an array to avoid repeating the extensive calculations for every user's entry.

If the nearest parking lot is occupied, the program loops to check the next shortest distance lot until an unoccupied lot is found. Once a suitable parking lot is identified, the loop terminates, and the system instructs the ticket printer to print the parking lot number along with a route map on the parking ticket. Simultaneously, a signal is sent to the control system to mark the assigned lot as occupied. The barrier gate then opens to allow the user to enter the parking area.

This proposed project primarily aims to integrate a Graphical User Interface (GUI) into the parking zone control system. A dedicated part of the program will focus on GUI development, facilitated by NI LabVIEW. LabVIEW simplifies this process by allowing GUI components to be automatically generated in one window while programming is developed in another.

The designed GUI will provide a comprehensive overview of the parking zone floor plan, with each parking lot displayed alongside an indicator showing its availability status. The system also includes an option to reserve specific parking lots based on customer requests by excluding them from availability calculations. The GUI will acquire real-time sensor data to display the status of all parking lots, enabling operators to monitor the system effectively.

Additional features include the ability to display statistics such as the estimated number of vehicles in the queue, the probability of having zero customers in the queue, average service time, and the total number of cars that have entered the parking zone. These data points will be accessible for operators to record and analyse.

Figure 4 illustrates the GUI design used to present information to the parking administrator. Through the GUI, administrators can view graphs depicting parking lot vacancies, queue probabilities, server utilization, and the estimated number of queued vehicles, all calculated and updated in real time. Indicators on the GUI align with the physical parking lot indicators, with light green signifying availability and dark green indicating occupancy. Administrators can navigate between parking levels using the tabs at the top of the interface. The system includes functionality to stop monitoring immediately with a "Stop Monitoring" button or reset statistics using the "Reset Statistics" button, clearing previous data to restart monitoring. For modelling purposes, the number of parking lots has been reduced from 32 to 16 per level to streamline the model's structure and reduce its complexity.

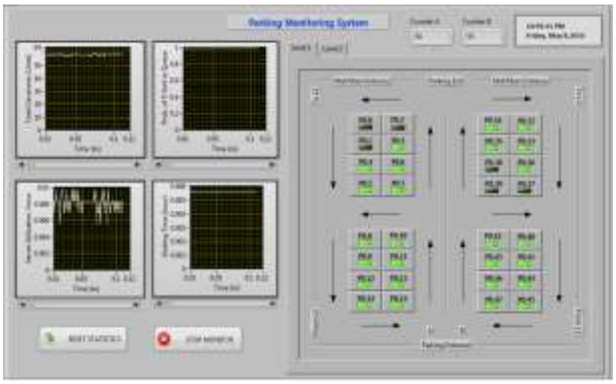

Figure 4. GUI for real-time monitoring

4. Results and Discussions

In the simulation GUI, both theoretical and simulated values are displayed on the graphs for easy comparison, with white plots representing theoretical values and red plots representing simulated values. The results for the exit gates are shown after the simulation is completed.

Analysis of simulation results across various scenarios reveals that system parameters are highly dependent on the arrival rate. For example, during weekends in non-peak hours, as illustrated in Figure 5, the system exhibits numerous vacancies due to the low arrival rate during these periods. The probability of having zero vehicles in the queue is approximately 0.87, indicating that the queue is empty most of the time. Server utilization during this period is around 0.069, meaning servers are active for about 248 seconds over a one-hour (3600 seconds) simulation. The estimated number of vehicles in the queue is approximately 0.001, which corresponds to an average of one vehicle entering the queue every 100 seconds.

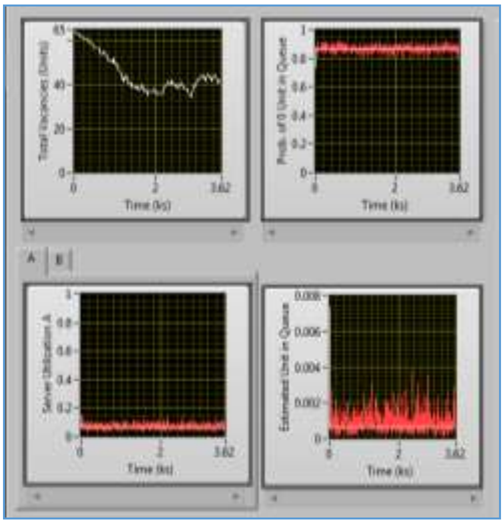

Figure 5. Entrance results for Saturday 8am to 9am simulation

During weekends at peak hours, as shown in Figure 6, the system has fewer vacancies compared to non-peak hours. The probability of having zero vehicles in the queue decreases to about 0.735, suggesting a higher likelihood of vehicles waiting in the queue. Server utilization increases to 0.153, indicating that the servers are active for approximately 551 seconds out of the one-hour simulation. The estimated number of vehicles in the queue rises to around 0.007, which translates to an average of one vehicle entering the queue every 14 seconds.

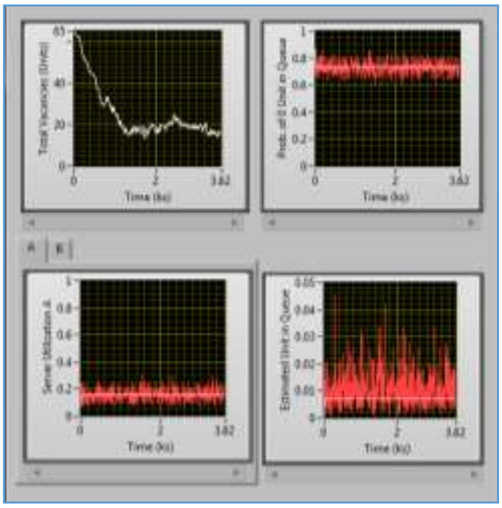

Figure 6. Entrance results for Sunday 1pm to 2pm simulation

In a randomly selected time simulation, as shown in Figure 7, the system is not fully occupied throughout the simulation period. The probability of having zero vehicles in the system is approximately 0.823, representing an average between peak and non-peak hours. Server utilization is around 0.097, equivalent to 349 seconds of busy time during the simulation. The estimated number of vehicles in the queue is 0.002, indicating one vehicle entering the queue every 50 seconds.

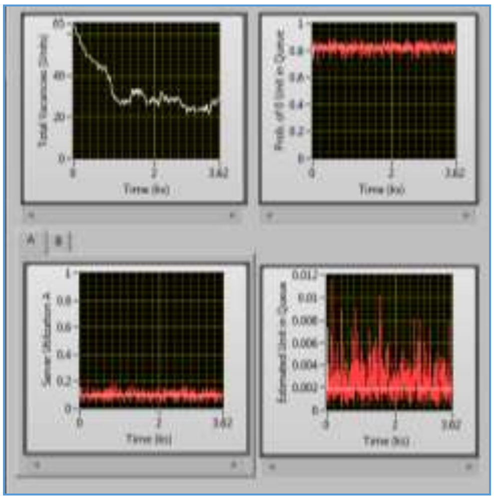

Figure 7. Entrance results for Saturday, 7pm to 8pm simulation

On weekdays during peak hours, as shown in Figure 8, the probability of having zero vehicles in the queue is approximately 0.756. Server utilization increases to 0.139, meaning the servers are busy for about 500 seconds out of one hour. The estimated number of vehicles in the queue is 0.005, equivalent to one vehicle arriving every 20 seconds.

During non-peak hours on weekdays, illustrated in Figure 9, the probability of having zero vehicles in the queue rises to approximately 0.895. Server utilization drops to 0.056, corresponding to 202 seconds of busy time, and the estimated number of vehicles in the queue is 0. This suggests that no queue is likely to form, as servers remain idle due to the low arrival rate.

At a randomly selected time on weekdays, as depicted in Figure 10, the probability of having zero vehicles in the queue is around 0.8. Server utilization is 0.111, or 400 seconds of busy time during the simulation, while the estimated number of vehicles in the queue is 0.004, meaning one vehicle arrives approximately every 25 seconds.

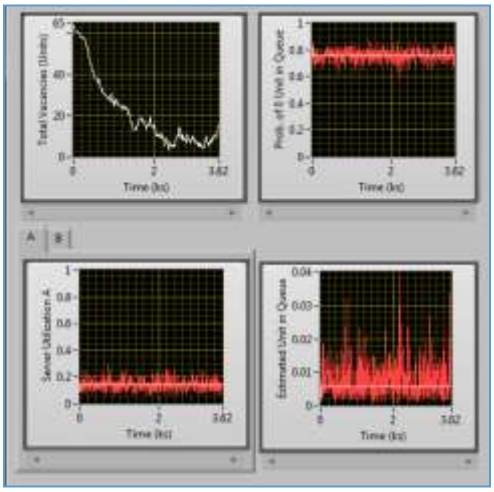

Figure 8. Entrance results for Tuesday, 9am to 10am simulation

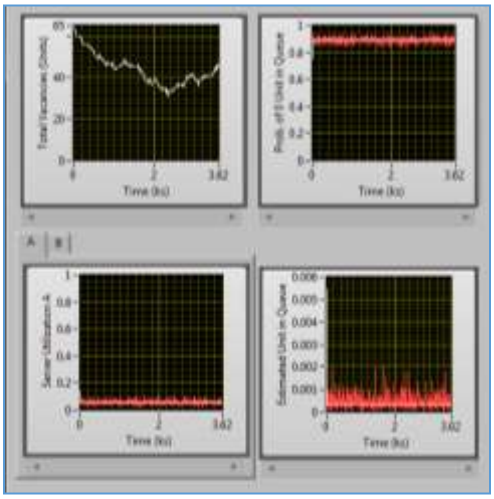

Figure 9. Entrance result for Wednesday, 5pm to 6pm simulation

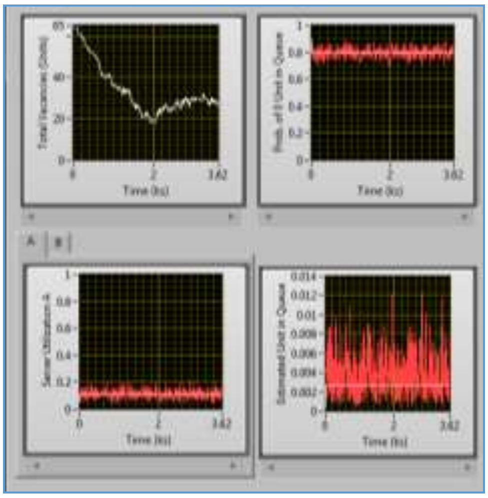

Figure 10. Entrance results for Friday, 9pm to 10pm simulation

Based on the analyzed data, it is evident that the system operates effectively under both low and high arrival rates, as there are no instances where the estimated number of vehicles in the queue or server utilization rises to abnormally high levels during the simulations. Additionally, as shown in Tables 1 to 3, the use of a Poisson random distribution in the M/M/2 queue system accurately simulates events. The differences between the simulation results and theoretical values for all parameters are minimal, further validating the system's accuracy.

Analysis of the data collected for the exit gate, which operates under a G/G/2/64 queue system, demonstrates that the system is stable and suitable for practical use. As illustrated in Figures 11 to 13 for selected scenarios, the servers at the exit gate exhibit significantly lower utilization compared to those at the entrance gate. This is because vehicles arriving at the exit gate are not influenced by external factors, as users can leave the mall at any time they choose.

The queue capacity is capped at 64 vehicles, matching the total number of parking spaces in the system. Due to the scattered nature of customer departures, queues rarely form at the exit gate. This explains the high probability of having zero vehicles in the queue, as shown in the figures. Additionally, the likelihood of multiple vehicles exiting simultaneously is extremely low.

Across all tested scenarios, the system remains stable, with server utilization and the probability of having zero vehicles in the queue consistently within the expected range of 0 to 1. The estimated queue size never reaches excessively high values, indicating the absence of long queues and further validating the system's reliability.

Table 1. Server utilization

Day	Time	Server Utilization				
		Theoretical	Simulation		Error (%)	
			Server A	Server B	Server A	Server B
6	8	0.069	0.071	0.070	2.899	1.449
7	13	0.153	0.156	0.157	1.961	2.614
6	19	0.097	0.097	0.100	0.000	3.093
2	9	0.139	0.140	0.141	0.719	1.439
3	17	0.056	0.058	0.055	3.571	1.786
5	21	0.111	0.114	0.113	2.703	1.770

Table 2. Probability of having zero units in queue

Day	Time	Prob. of Zero Unit in Queue		
		Theoretical	Simulation	Error(%)
6	8	0.870	0.869	0.115
7	13	0.735	0.732	0.408
6	19	0.823	0.821	0.243
2	9	0.756	0.758	0.265
3	17	0.895	0.894	0.112
5	21	0.800	0.802	0.250

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Table 3. Estimated queue content

Day	Time	Estimated Queue Content		
		Theoretical	**Simulation**	**Error(%)**
6	8	0.001	0.001	0
7	13	0.007	0.007	0
6	19	0.002	0.002	0
2	9	0.005	0.005	0
3	17	0.000	0.000	0
5	21	0.003	0.003	0

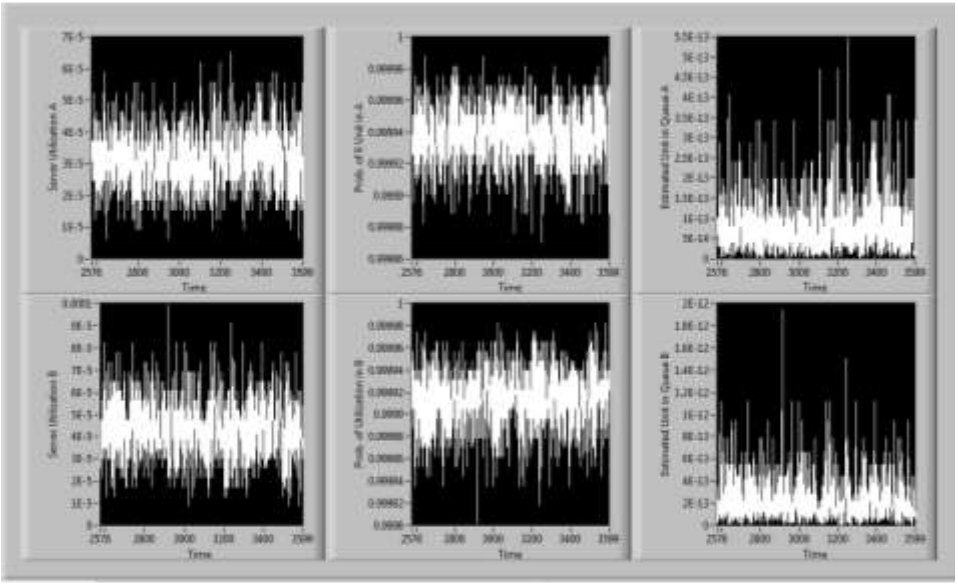

Figure 11. Exit results for Saturday, 7pm to 8pm simulation

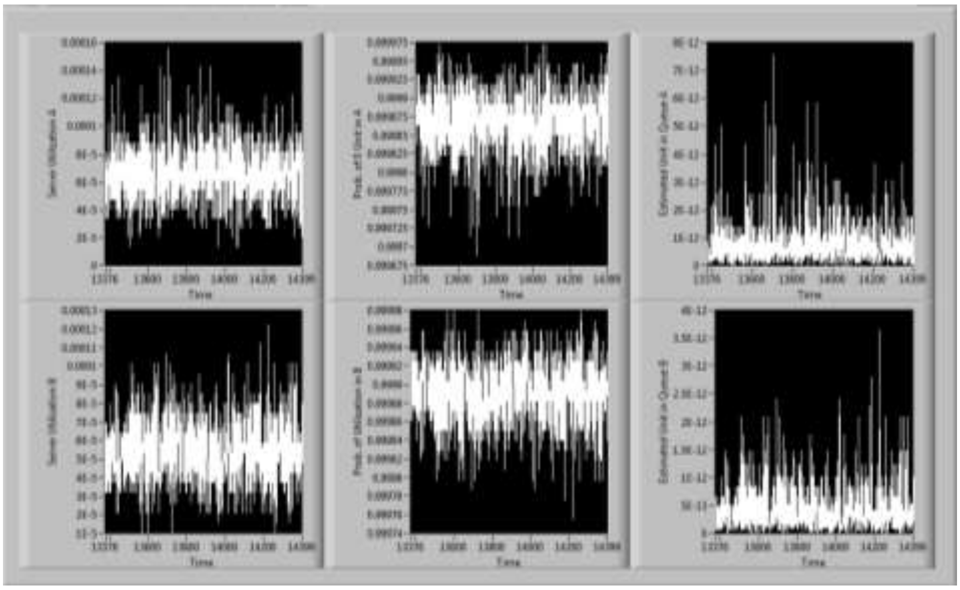

Figure 12. Exit results for Sunday, 1pm to 2pm simulation

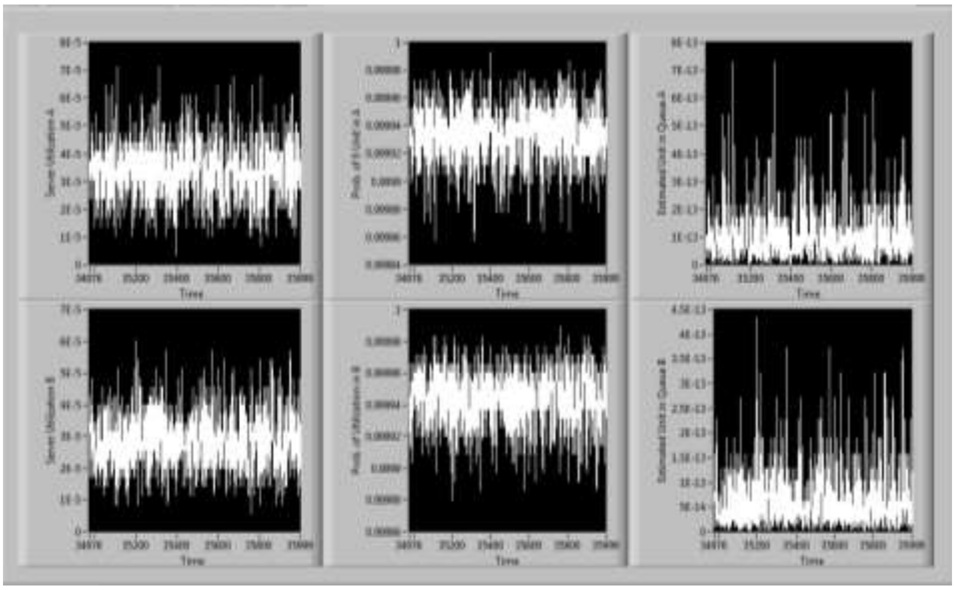

Figure 13. Exit result for Wednesday, 5pm to 6pm simulation

5. CONCLUSIONS

The proposed system has been designed to meet the requirements effectively. For modeling the parking system, the M/M/2 queue model was selected for the entrance gate, as it accurately represents the arrival rate and service time distribution at that point. For the exit gate, the G/G/2/64 model was chosen because it captures the random and independent arrival and departure patterns of users at the exit. Through this modeling approach, key probabilities, such as the likelihood of having zero vehicles in the system or a specific number of vehicles in the queue, can be calculated using mathematical models. Additionally, performance metrics including the expected average queue length, number of vehicles in the system, total time spent, and waiting time can be determined using established formulae, providing valuable insights into system performance. In conclusion, leveraging queue theory principles, a smart parking system GUI has been designed. This GUI is intended for implementation in conventional smart parking systems, enabling operators to monitor their parking zones with ease. The simulation results validate that the system adheres to queue theory principles, demonstrating its ability to accurately represent real-world parking scenarios.

6. ACKNOWLEDGMENTS

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