

# Influence of Unsupported Sleepers on the Wheel-Rail Interaction

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**Abstract:** This paper explores the influence of unsupported sleepers on the interaction between wheels and rails in railway systems. Sleepers, also known as ties, play a crucial role in providing support and stability to the rail infrastructure. However, when sleepers become unsupported or deteriorate, they can significantly impact the dynamic behavior of trains and tracks. Through a combination of analytical modeling and numerical simulations, this study investigates the effects of unsupported sleepers on various aspects of wheel-rail interaction forces. The findings highlight the importance of proactive maintenance strategies to address sleeper support issues and maintain the reliability of railway networks.

**Keywords:** unsupported sleeper, uneven settlement, wheel-rail interaction, train-track interaction, track dynamics

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## 1. INTRODUCTION

The interaction between wheels and rails constitutes a pivotal aspect of railway infrastructure, significantly influencing safety and operational efficiency [1]. The sleepers are crucial components that provide essential support to the rails, ensuring the stability of the track structure. However, the presence of unsupported or deteriorating sleepers poses notable challenges to this interaction, impacting various facets of railway operations [2]. It has been demonstrated by Naseri and Mohammadzadeh [3] that when a sleeper-group is unsupported within the 2/8 and 5/8 of a span, the bridge is at its maximum acceleration. Additionally, the bridge's deck is loaded up to 60% more than it was before. Unsupported sleepers can induce alterations in track geometry, leading to uneven settling and rail misalignment, which not only compromise passenger comfort but also accelerate wear and tear on track components, necessitating increased maintenance efforts [4]. Moreover, the loss of support from deteriorating sleepers can trigger dynamic instabilities, exacerbating track irregularities and oscillations, thus compromising safety and ride quality, particularly at higher speeds and on curved sections of track. Furthermore, unsupported sleepers can impede proper drainage and ballast retention, exacerbating track deterioration and maintenance requirements. Addressing such challenges necessitates proactive maintenance strategies to mitigate the adverse effects of unsupported sleepers on the wheel-rail interaction, ensuring the continued safety, efficiency, and sustainability of railway networks. Recent studies have investigated the effects of unsupported sleepers on the dynamic behavior of railway tracks, revealing significant implications for track maintenance and operation safety. Grassie and Cox [5] observed increased deflections of fastening systems and sleeper strains in tracks with unsupported sleepers, emphasizing the importance of ballast presence. Kaewunruen and Remennikov [6] employed finite element simulation and field experiments to study track settlement due to sleeper hanging. Zhu et al. [7] conducted numerical analyses to examine the dynamic response of wheel-rail components in the presence of unsupported sleepers, highlighting the influence of various factors on impact loads. In this context, this paper aims to further explore and elucidate the impact of unsupported sleepers on wheel-rail interaction dynamics. Through a combination of analytical modeling, numerical simulations, and empirical investigations, this study endeavors

to provide insights into the effects of unsupported sleepers on track performance, safety, and maintenance requirements. By comprehensively understanding these effects, railway authorities can develop informed maintenance strategies and optimize track design to ensure the integrity and efficiency of railway operations.

## 2. SIMULATION OF TRAIN-TRACK INTERACTION

Simulation of train-track interaction is a vital aspect of railway engineering, providing insights into the complex dynamics and behaviors of trains traversing along the track infrastructure [8]. This section explores the significance of simulation techniques in understanding and optimizing the interaction between trains and tracks, highlighting the methodologies employed, key parameters considered, and the benefits of simulation-based approaches.

### 2.1 Model description

The three-layered track model incorporates the essential components of a railway structure, namely rails, sleepers (or ties), and ballast. The rails serve as the primary load-bearing elements, supporting the weight of the train and transmitting forces to the sleepers. The sleepers, positioned at regular intervals along the track, provide lateral and longitudinal support to the rails and distribute the load to the underlying ballast layer. The ballast layer, composed of crushed stone or other aggregate materials, serves to stabilize the track, provide drainage, and distribute loads to the subgrade.

The moving multibody dynamic simulation approach employs sophisticated numerical techniques to model the interaction between the train and track components in a dynamic environment. This approach considers the motion and forces acting on each individual component of the train and track system. It accounts for factors such as vehicle dynamics, wheel-rail contact mechanics, track irregularities, and environmental conditions. By integrating these factors into a unified simulation framework, the model can efficiently predict the dynamic response of the system under various operating conditions, including different train speeds, loads, and track configurations.

The equations governing the motion of the train and track components in the multibody dynamic simulation can be expressed as follows:

### 2.1.1 Equations of Motion for Train Components

The train dynamics are modeled using a structured approach consisting of three major components: a car-body, two bogies, and four wheelsets for each railcar. These constituents are interconnected through linear spring-damper systems. The car-body is delineated as a rigid beam, with the exclusion of bending mode shapes. It is affixed to both the rear and front bogies via a secondary spring-damper arrangement. The bogies, mirroring the characteristics of the car-body model, possess two degrees of freedom and are tethered to the wheels through the primary spring-damper mechanism. Consequently, the total degrees-of-freedom (DOFs) for a vehicle are determined to be 10. The detail of this model is explained in detail in [9]. The schematic depiction of this streamlined vehicle model is presented in Figure 1. The dynamic equations of the train model can be expressed as follows:

$$[M_v]\{\ddot{u}_v\}+[C_v]\{\dot{u}_v\}+[K_v]\{u_v\}=\{F(t)\} \quad (1)$$

Where M, C, K, and F are mass, damping, stiffness, and force, respectively.

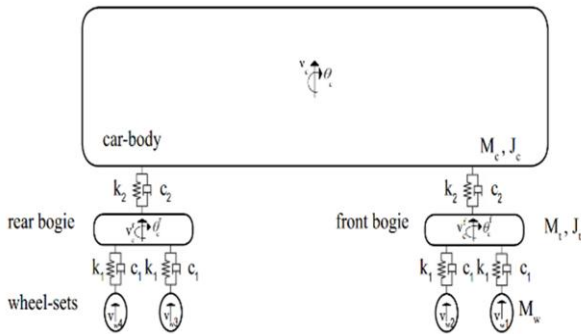


Figure. 1 Schematic of train model

### 2.1.2 Equations of Motion for Track Components

A beam-sleeper-ballast component model was used to simulate the track system. This model utilizes a continuous Euler-Bernoulli beam positioned atop a two-layer mass-spring system at specified intervals. Within this framework, both the sleeper and ballast elements are characterized by multi-rigid-body dynamics featuring elastic connections, with vertical displacement being the sole consideration. To account for the coupling effects among ballast elements, such as shear interlocking, the proposed method have included a pair of shear stiffness parameters and shear damping coefficients between adjacent ballast elements using the developed model explained in [10]. The schematic representation of the track model is depicted in Figure 2.

$$\begin{bmatrix} M_r & 0 & 0 \\ \text{sym.} & M_s & 0 \\ & & M_b \end{bmatrix} \begin{Bmatrix} \ddot{u}_r \\ \ddot{u}_s \\ \ddot{u}_b \end{Bmatrix} + \begin{bmatrix} C_r & C_{r/s} & 0 \\ \text{sym.} & C_s & C_{s/b} \\ & & C_b \end{bmatrix} \begin{Bmatrix} \dot{u}_r \\ \dot{u}_s \\ \dot{u}_b \end{Bmatrix} + \begin{bmatrix} K_r & K_{r/s} & 0 \\ \text{sym.} & K_s & K_{s/b} \\ & & K_b \end{bmatrix} \begin{Bmatrix} u_r \\ u_s \\ u_b \end{Bmatrix} = \begin{Bmatrix} F_r(t) \\ F_s \\ F_b \end{Bmatrix} \quad (2)$$

Where M, C, K, and F are mass, damping, stiffness, and force, respectively. The subscripts of r, s, and b present rail, sleeper and ballast, respectively. Additionally, u is the response, and its derivatives are denoted by dots.

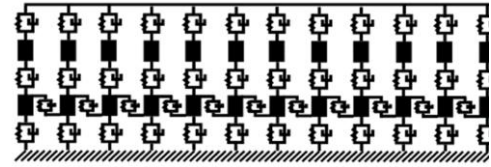


Figure. 2 Schematic of track model

### 2.1.3 Equations of Motion for wheel-rail interactions

According to the Hertzian contact theory, the interaction force between wheel and rail at location can be expressed as [3]:

$$F = \begin{cases} cy\sqrt{y} & y \leq 0 \\ 0 & y > 0 \end{cases} \quad (3)$$

Where y represents the relative displacement of the wheelset and rail. c is the nonlinear Hertzian contact coefficient.

### 2.1.4 Unsupported sleeper model

In this study, the unsupported sleeper is represented by incorporating a gap size beneath the sleeper, as depicted in Figure 3.

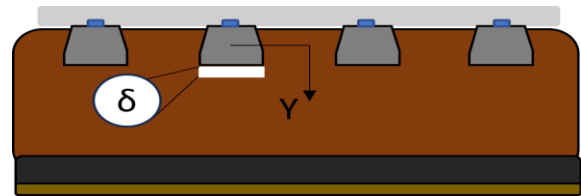


Figure. 3 Schematic of unsupported sleeper

Utilizing this model, the interaction between the sleeper and the ballast is depicted as:

$$f = \begin{cases} k(Y - \delta) & Y \leq \delta \\ 0 & Y > \delta \end{cases} \quad (4)$$

Where k, Y and delta ballast stiffness, sleeper deflection, and the gap size, respectively. The behavior of simulated stiffness between sleeper and ballast is demonstrated in Figure 4 for supported and unsupported sleepers.

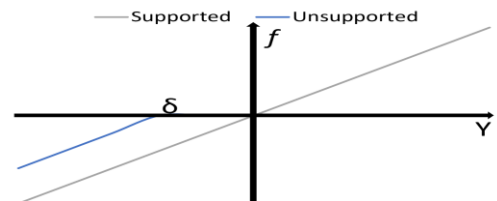


Figure. 4 Force-deflection graph of the sleeper-ballast interaction

## 2.2 Train-track coupled system

Modeling the interaction between the train and track components as a coupled system involves considering the dynamic response of both the train and the track to the forces exerted during motion. By solving these coupled equations of motion numerically using appropriate integration algorithms, the proposed method can be implemented to simulate the dynamic behavior of the entire train-track system over time and analyze its response to various operating conditions and environmental factors.

In this study, the dynamic equations of both subsystems are solved based on Newmark's finite difference scheme, which is

widely used in engineering practices [9]. Initially, the condition of the whole system is defined and then, the next step's responses are calculated using the Newmark method.

### 3. VERIFICATION

This study was designed to assess the robustness and accuracy of the proposed model by comparing its outcomes with those presented in [11]. This reference serves as a benchmark for evaluating the performance of the model, particularly in simulating the intricate dynamics of wheel-rail interaction. The results of this validation process are meticulously illustrated in Figure 5, where a detailed comparison between the two models is shown. Notably, the graphical representation exhibits a remarkable level of agreement between the simulated outcomes, indicating a high degree of consistency and concordance. This corroborative evidence serves to reinforce the confidence in the proposed method to accurately capture and analyze the intricate nuances of wheel-rail contact force dynamics

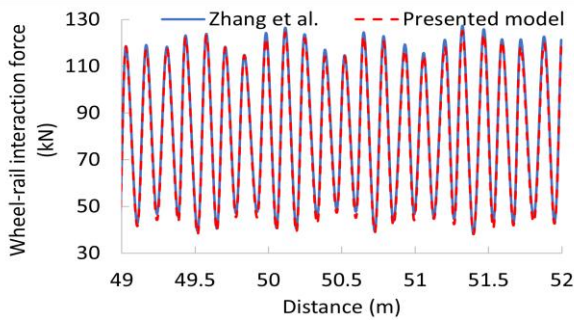


Figure. 5 Model verification with the benchmark results

### 4. Results and discussions

The presented model is used to investigate deviation in the wheel-rail interaction due to the number of unsupported sleepers, gap size and train speed. For the first two cases the train speed is 80 km/h. the details of each set of analysis are discussed in follow:

#### 4.1 Effects of the number of unsupported sleepers

Figure 6 illustrates the relationship between the wheel-rail interaction force and the number of unsupported sleepers. In this analysis, the dynamic force results for up to 8 consecutive unsupported sleepers are compared with the scenario where there is no gap between the sleeper and ballast surface. In the case of no gap sleeper, the interaction force between the wheel and rail is measured at 120 kN. As the number of unsupported sleepers increases, the dynamic load also increases. Specifically, when only one sleeper is unsupported in the track, the dynamic load increases by 2.5%. This deviation escalates to 6.7%, 10%, 14.2%, 19.2%, 23.3%, 26.7%, and 29.2% as the number of unsupported sleepers increases from 2 to 8, respectively.

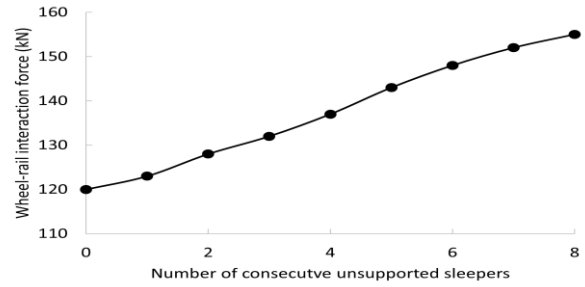


Figure. 6 Wheel-rail interaction force variation with respect to the number of unsupported sleepers

#### 4.2 Effects of the number of gap size

Figure 7 illustrates the variation in wheel-rail interaction forces for different gap sizes between the sleeper and ballast. It is evident that the dynamic force increases rapidly with larger gap sizes. The results indicate that for every 1 mm increase in the gap size, the amplitude of the wheel-rail interaction force is enhanced by approximately 10 percent. For instance, in the scenario with a 1 mm gap, the dynamic force increases by approximately 10% from 120 kN to 131 kN. This trend continues, leading to an increase in force to approximately 147 kN and 166 kN for gap sizes of 2 mm and 3 mm, respectively.

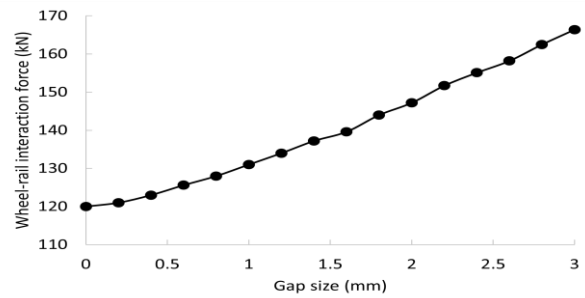


Figure. 7 Wheel-rail interaction force variation with respect to the gap size

#### 4.3 Effects of the number of train speed

Figure 8 depicts the alterations in wheel-rail interaction forces as the train accelerates from 60 to 200 km/h. The impact of unsupported sleepers on dynamic force becomes more apparent as the train operates at higher speeds. The findings reveal that as the speed escalates from 60 to 200 km/h, the wheel-rail interaction forces escalate from 114.8 to 159.6 kN. This corresponds to an approximately 39% increase in dynamic force.

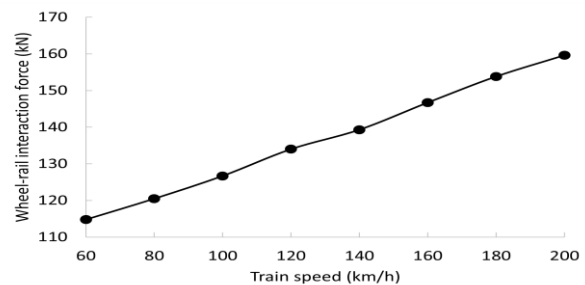


Figure. 8 Wheel-rail interaction force variation with respect to the train speed

## 5. CONCLUSION

This paper has investigated the significant impact of unsupported sleepers on the intricate dynamics of wheel-rail interaction within railway systems. Analytical modeling and numerical simulations have been used to explore how deterioration or lack of support in sleepers can profoundly affect train and track behavior. The findings highlight the critical importance of proactive maintenance strategies aimed at addressing sleeper support deficiencies to ensure the continued reliability and safety of railway networks.

Moving forward, further research could delve into more complex aspects of sleeper support, such as the effects of varying degrees of support deficiency on different types of trains and track configurations. Additionally, exploring innovative solutions for enhancing sleeper support and mitigating the adverse effects of unsupported sleepers could pave the way for a more resilient railway infrastructure.

## 6. REFERENCES

- [1] Yang, Y., et al., Wheel/rail dynamic interaction induced by polygonal wear of locomotive wheels. *Vehicle System Dynamics*, 2022. 60(1): p. 211-235. DOI: <https://doi.org/10.1080/00423114.2020.1807572>
- [2] Liu, J., et al., Influence of unsupported sleepers on the dynamic stability of ballasted bed based on wheelset impact tests. *Railway Engineering Science*, 2023. 31(1): p. 52-60. DOI: <https://doi.org/10.1007/s40534-022-00292-z>
- [3] Naseri, R., and S. Mohammadzadeh, Nonlinear Train-Track-Bridge Interaction with Unsupported Sleeper Group. *International Journal of Railway Research*, 2020. 7(1): p. 11-28. DOI: <https://doi.org/10.22068/IJRARE.7.1.11>
- [4] Yang, F., et al., Wheel-rail rolling contact behavior induced by both rail surface irregularity and sleeper hanging defects on a high-speed railway line. *Engineering Failure Analysis*, 2021. 128: p. 105604. DOI: <https://doi.org/10.1016/j.engfailanal.2021.105604>
- [5] Grassie, S. and S. Cox, The dynamic response of railway track with unsupported sleepers. *Proceedings of the Institution of Mechanical Engineers, Part D: Transport Engineering*, 1985. 199(2): p. 123-136. DOI: [https://doi.org/10.1243/PIME\\_PROC\\_1985\\_199\\_149\\_01](https://doi.org/10.1243/PIME_PROC_1985_199_149_01)
- [6] Kaewunruen, S. and A. Remennikov, Investigation of free vibrations of voided concrete sleepers in railway track system. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2007. 221(4): p. 495-507. DOI: <https://doi.org/10.1243/09544097JRRT141>
- [7] Zhu, J.J., et al., Development of a vehicle-track model assembly and numerical method for simulation of wheel-rail dynamic interaction due to unsupported sleepers. *Vehicle system dynamics*, 2010. 48(12): p. 1535-1552. DOI: <https://doi.org/10.1080/00423110903540751>
- [8] Naseri, R., Consequences of rail surface local irregularities on railway concrete bridge vibrations. 2020, Iran University of Science and Technology (IUST).
- [9] Naseri, R., S. Mohammadzadeh, and D.C. Rizos, Rail surface spot irregularity effects in vehicle-track interaction simulations of train-track-bridge interaction. *Journal of Vibration and Control*, 2024: p. 10775463241232024. DOI: <https://doi.org/10.1177/10775463241232024>
- [10] Heydari, H., N. Khanie, and R. Naseri, Formulation and Evaluation of the Ballast Shear Interlocking Coefficient based on Analytical, Experimental, and numerical Analyses. *Construction and Building Materials*, 2023. 406: p. 133457. DOI: <https://doi.org/10.1016/j.conbuildmat.2023.133457>
- [11] Zhang, S.-m., et al., Dynamic wheel-rail interaction at high speed based on time-domain moving Green's functions. *Journal of Sound and Vibration*, 2020. 488: p. 115632. DOI: <https://doi.org/10.1016/j.jsv.2020.115632>