Research on the Working Characteristics of Solid-Fuel Rocket Engines

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Abstract: This article presents a method for building a mathematical model to determine the working parameters of a solid-fuel rocket engine. By changing some structural parameters around the initial values, the working characteristics of the rocket engine were investigated to evaluate the reliability of the designed structural parameters. The C5K solid-fuel rocket engine was selected to verify the model. The survey results show that: the volume of the combustion chamber increases by 20%, pressure decreases by 1.07%, thrust decreases by 1.07%, and engine working time increases by 2.9%. In addition, the change in critical cross-sectional diameter and the length of the propellant rod also significantly affects the thrust and working time of the engine. The results of the article are used as a theoretical basis to calculate and design rocket engines to ensure high accuracy.

Keywords: Rocket engine, solid fuel, thrust, combustion chamber, C5K

1. INTRODUCTION

Solid-fuel rocket engines working based on chemical fuels are characterized by complex physicochemical processes taking place inside the engine's combustion chamber and nozzles, see Figure 1. The working process of the solid fuel rocket engine begins when the primer device works. Burned primer creates a stream of combustion products with high velocity and temperature, increasing the temperature and pressure in the combustion chamber. Then the free volume in the combustion chamber is filled with very high-temperature combustion products. The temperature of the combustion products of the primer dose is transferred to the surfaces of the propellant dose through convection and radiation heat transfer. When the propellant reaches the temperature and temperature gradient to critical conditions, the propellant is ignited, creating combustion products with high temperature and pressure. Combustion products eject through the nozzle at a speed greater than the speed of sound, creating thrust for the rocket to move (the direction of thrust is opposite to the direction of airflow).

Solid fuel rocket engines have simple structures, high reliability, are simple to use, and are always in a state of combat readiness. Solid-fuel rocket engines can create large thrusts in a short time. , the cost of manufacturing solid-fuel rocket engines is much lower than other types of liquid-fuel rocket engines. However, solid fuel rocket engines also have the following disadvantages: the value of thrust depends heavily on the initial temperature of the propellant, thrust control, and complex thrust programming, and manufacturing cost is higher than liquid fuel, the lower pressure limit ensures relatively high stable combustion. With such outstanding advantages, solid-fuel rocket engines are widely used in military and aerospace around the world.

There are many studies on solid-fuel rocket engines in the world, and many studies have shown the important working characteristics of solid-fuel rocket engines. Research [1] has determined the working parameters of a solid-fuel rocket engine, however, this study only calculated for a specific engine, without research on the structural characteristics of the engine. Research [2] only focuses on determining the movement parameters of gas flow through the nozzle of a solid-fuel rocket engine. Research [4-6] has mainly studied thermal protection solutions for rocket engines. In addition,

studies [7,8] have also mentioned the working parameters of rocket engines, however, no studies have evaluated the influence of the structural parameters of the engine on the working ability of solid-fuel rocket engines.



Figure 1. The general structure of the solid-fuel rocket engine Within the framework of the article, the authors focused on researching and building a mathematical model to determine the working parameters of solid fuel rocket engines such as pressure in the combustion chamber p, thrust of the P motor, and t motor working time. Numerical methods were used to solve the problem in this study. Mathematical modeling is applied to determine the working parameters of the C5K solid-fuel rocket engine. The results obtained from this research can be used as a theoretical basis for calculating and designing rocket engines to ensure high accuracy.

2. SET UP THE MATHEMATICAL MODEL

Solid fuel rocket engines include the following main parts: Combustion chamber, propellant, nozzle, propellant base, and primer device. In particular, the combustion chamber of the solid fuel rocket engine has a cylindrical shape, one end is covered and connected to the combat part and the other end is attached to the nozzle block. The combustion chamber is the basic device of the engine in which the process of converting the chemical energy of the fuel into the kinetic energy of the airflow in the nozzle occurs. Because the airflow moves from the nozzle out at high speed, it creates thrust to push the rocket into motion. Therefore, the value of specific thrust, working reliability, structural mass, etc. depends greatly on the level of completion of the combustion chamber and nozzles. To study the working characteristics of solid-fuel rocket engines, specific problems need to be established and calculated such as the interior ballistic problem and the problem of gas flow movement through the nozzle.

2.1 Set up the interior ballistic problem

a) Assumptions

The introduction of assumptions is intended to simplify the complex processes that occur in the combustion chamber of a rocket engine. To solve the basic problem of interior ballistic by theoretical methods, the influence of other factors is taken into account by the experiment coefficient. The following assumptions are used to solve the interior ballistic problem of a solid-fuel rocket engine, see in [9-12]:

- The igniter ignites instantaneously, then at t = 0, $p = p_{in}$;

- The propellant burns according to the laws of geometry;

- Static pressure does not change along the combustion chamber and is equal to the braking pressure at the nozzle outlet;

- The temperature of the combustion gas in the combustion chamber T_0 does not change and is equal to:

$$T_0 = \frac{T_1}{k}$$

- Heat loss in the combustion chamber is given by the factor χ_n :

$$\chi_n = 1 - \frac{a}{1 + b\psi}$$

- Neglecting the volume of the gas particles themselves: $\alpha = 0.$

- The geometrical parameters of the motor do not change during engine operation.

b) System of interior ballistic differential equations for solid fuel rocket engines

- The stage where propellant burns in the combustion chamber

The basic system of equations for the interior ballistic of solid fuel rocket engines is established based on thermodynamic and aerodynamic relationships, describing the state of the medium in the combustion chamber and nozzle, and the dimension characteristics of the shape of the propellant.

The law of change of combustion gas pressure over time p(t) is determined from the gas mass balance equation at each time. The gas flow rate is equal to the total airflow through the nozzle and the remaining gas volume in the combustion chamber:

$$\dot{m}_{+} = \dot{m}_{-} + \dot{m} \tag{1}$$

Where:

 $\dot{m}_{+} = Su\rho_{T}$: combustion gas flow is generated;

$$\dot{m}_{-} = \frac{\varphi_2 K_0(k) F_{th} p}{\sqrt{\chi R T_0}}$$
: combustion gas flow through the nozzle;

 $\dot{m} = \frac{dm}{dt}$: remaining gas flow in the combustion chamber;

The variation of remaining gas flow in the combustion chamber at any time is determined by the equation of state:

$$pV = m\chi_n RT_0 \tag{2}$$

Where:

S: burning surface area;

u: burning speed of the propellant;

 ρ_{T} : density of propellant;

 φ_2 : flow loss coefficient;

 $K_0(k)$: Function of the adiabatic exponent;

 F_{th} : throat cross-sectional area;

R: gas constant;

V: volume of gas displaced in the combustion chamber at the time of survey;

m: mass of gas in the combustion chamber;

Then the expression for m has the form:

$$\dot{m} = \frac{dm}{dt} = \frac{d}{dt} \left[\frac{pV}{\chi_n RT_0} \right] = \frac{1}{RT_0} \frac{d}{dt} \left[\frac{pV}{\chi_n} \right]$$
(3)

Substituting expressions (2), (3), (4) into expression (5), the law of pressure change over time p(t) is determined by the following expression:

$$\frac{dp}{dt} = -\frac{1}{V} \left[\left(\varphi_2 K_0(k) F_{th} \sqrt{\chi_n f_0} + Su - V \chi_1 \right) p - Su \chi_n f_0 \rho_T \right]$$
(4)

Where:

 $f_0 = RT_0$: Isobaric propellant force:

$$\chi_1 = \frac{1}{\chi_n} \frac{d\chi_n}{dt}$$

$$V = V_k - \frac{\omega}{\rho_T} (1 - \psi)$$
: Free volume of gas at each instant of

time;

 V_k : Combustion chamber volume;

 ω : Mass of propellant;

 ψ : relative mass of burning propellant;

The law of changing heat loss χ_n in the combustion chamber can be written as:

$$\chi_n = 1 - \frac{a}{1 + b\psi} \tag{5}$$

Where: *a*, *b*: experimental coefficients;

The law of change of relative propellant mass ψ has the form:

$$\frac{d\psi}{dt} = \frac{\rho_T S u}{\omega} \tag{6}$$

 $\frac{d\psi}{dt}$ - gas generation rate.

The system of differential equations determines the rules for changing over time the pressure p(t), the relative mass of propellant burned ψ (t), heat loss in the combustion chamber

 $\chi_n(t)$ from the time the engine starts working (t_0) until the time the propellant burns out (t_k) is described by the following equation system:

$$\begin{cases} \frac{d\psi}{dt} = \frac{Su\rho_T}{\omega_T} \\ \frac{d\chi_n}{dt} = \frac{ab}{(1+b\psi)^2} \frac{d\psi}{dt} \\ \frac{dp}{dt} = -\frac{1}{V} [(\varphi_2 K_{0(k)} F_{th} \sqrt{\chi_n f_0} + Su - V\chi_1) p - Su\chi_n f_0 \rho_T] \end{cases}$$
(7)

Where: $u = u_1 p^{\nu} f_1(T_{in}) \varphi(w)$ burning speed of the propellant.

 u_1 : burning rate coefficient;

 υ : Exponential index in the law of fire rate;

S = S(t): burning surface area of the discharged dose;

The temperature function is determined according to Eq:

 $f_1(T_{in}) = \left[1 - K_T(T_{in} - T_c)\right]^{-1}$ Where:

 K_{τ} : Thermal stability coefficient;

 T_{in} : initial temperature of the propellant;

 T_c : temperature under standard conditions.

The erosion function of the burning rate law is determined according to the following expression:

$$\varphi_w = 1 + K_w w^2 \tag{9}$$

Where: K_{w} : erosion function coefficient;

w =
$$\frac{\varphi_2 K_0(k) \sqrt{\chi_n f_0}}{F_{td}}$$
: velocity of the airflow;

Initial conditions: $t=t_0$; $\psi=\psi_0$; $p=p(t_0)=p_m$;

 $\chi_n = \chi_n(t_0).$

- The stage where the combustion gases move freely

Suppose by the time t_k the propellant burns out, The pressure of the gas in the combustion chamber is p_k . Khối lượng khí mk The mass of gas in the combustion chamber is determined directly from the equation of state, [6]:

$$m_k = \frac{p_k N_k}{f_0} \tag{8}$$

The specific volume of gas V_k in the combustion chamber at time tk is determined from the expression:

$$v_k = v(t_k) = \frac{V_k}{m_k} = \frac{f_0}{p_k}$$
(9)

Gas pressure in the combustion chamber at this stage:

$$p = p_k \cdot (1 + Bt)^{-\frac{2k}{k-1}}$$
(10)

Where,
$$B = \frac{k-1}{2.k} b$$
, $b = \frac{\phi_2 \cdot K_0(k) \cdot k \cdot F_{th}}{m_k} \sqrt{\frac{p_k}{v_k}}$.

2.2 ESTABLISH THE FORMULA TO DETERMINE THE THRUST OF A SOLID-FUEL ROCKET ENGINE

Thrust force P is generated by the movement of combustion gas through the nozzle at high speed. The direction of thrust is opposite to the direction of movement of the combustion gas flow. Thrust force P is the combined force of pressure forces inside and outside the engine.

The relationship between the thrust force and the characteristics of the system and the surrounding environment is determined from Euler's equation of motion (momentum conservation equation):

$$\frac{d(m.w)}{dt} = P \tag{11}$$

(12)

Where:

(8)

m - Mass of the survey system;

w - Speed of motion of the system;

P - The combined force acts on the system.

According to the law of conservation of momentum:

$$-P + (p_a - p_n).F_a = -m.w_a$$

$$\mathbf{P} = \mathbf{m}.\mathbf{w}_a + F_a(p_a - p_n) \tag{13}$$

Where

then

m - gas flow mass through the nozzle.

$$\overset{\bullet}{m} = \frac{\phi_2 \cdot K_0(k)}{\sqrt{RT_0}} \cdot F_{th} \cdot p \tag{14}$$

 $F_{\rm a}$ - outlet cross-sectional area of the nozzle.

$$w_{a} = \phi_{1} F_{w}(\xi_{a}, k) \sqrt{R.T_{0}}$$
(15)

 φ_1 - Coefficient of flow speed loss caused by friction and flow constriction (reducing the aerodynamic properties of the flow) $\varphi_1 < 1$;

 φ_2 - The loss factor includes losses caused by friction between the gas flow and the nozzle wall and the phenomena of flow interruption and looping ($\varphi_2 < 1$).

3. CALCULATION RESULTS AND COMMENTS

3.1 Input parameters

The working parameters of the solid-fuel rocket engine C5K are determined with the input parameters shown in Table 1, [13-17].

Parameters	Value	Unit				
Parameters of the rocket engine						
Inner diameter of the combustion chamber	0.054	m				
Outer diameter of the combustion chamber	0.056	m				
Length of the combustion chamber	0.375	m				
Diameter of the nozzle outlet	0.035	m				
Diameter of throat	0.012	m				
Diameter of nozzle inlet	0.054	m				
Number of nozzles	1	m				
Length of throat	0.017	m				
Nozzle wall thickness at the throat	0.0061	m				
Angle of inclination of the converging of the nozzle	50	deg				
Angle of inclination of the diverging of the nozzle	102	deg				
Parameters of the propellant						
Initial length of the propellant	0.349	m				
Outer diameter of the propellant	0.045	m				
Inner diameter of the propellant	0.0085	m				
Number of propellant bars	1					
Density of propellant	1600	kg/m ³				
Fire rate exponent	0.698					
Law coefficient of fire rate	1.2.10-7	m ³ /Ns				
Thermal stability coefficient of pressure	0.011	1/K				
Powder force	570000	J/kg				
Primer pressure	400000	Pa				
Erosion coefficient	0.000006	S ² /m ²				
Initial temperature of the propellant	293	^{0}K				

Table 1. Engine parameters and propellant of the C5K engine

The system of equations (8) is solved simultaneously with equation (12) using Matlab software with input parameters including structural parameters of the combustion chamber and propellant parameters in Table 1. Typical results are obtained as follows: Average pressure in the engine combustion chamber over time (see Figure 2); The law of engine thrust over time is shown in Figure 3.



Figure 2. Law of pressure in the engine combustion chamber over time



Figure 3. Law of thrust over time

3.2 Investigate the influence of combustion chamber diameter

The process of designing and manufacturing solid fuel rocket engines often has errors due to elastic deformation of the technological system or due to the accuracy and wear of fixture equipment. This is the cause of size errors compared to the design when manufacturing rocket engines. These errors will affect the engine's working parameters. Therefore, it is necessary to evaluate the influence of the engine's geometric parameters on the pressure and thrust of a solid-fuel rocket engine.

The influence of engine structural parameters such as combustion chamber volume and throat cross-sectional area was investigated on the same type of engine.

Table 2. Working parameters of the C5K engine

Rocket engine	Combustion chamber volume (m ³)	Throat cross- sectional diameter (m)	Maximum pressure value (MPa)	Maximum thrust value (N)	Engine working time (s)
C5K	9.165·10 ⁻⁴	0.0118	14.14	3500	0.624

To investigate the influence of the combustion chamber volume, the internal diameter of the combustion chamber is changed while maintaining the parameters of the throat crosssectional area of the nozzle and the geometrical characteristics of the propellant. The survey results are shown in Table 3.

Table 3. Table of pressure, thrust, and engine's working time

Volume of the combustion chamber (m ³)	Changes in volume parameter s (%)	Throat cross- sectional diameter (m)	Maximum pressure value (MPa)	Maximum thrust value (N)	Engine working time (s)
9.623·10 ⁻⁴	5	0.0118	14.08	3482	0.627
10.08.10-4	10	0.0118	14.03	3475	0.629
10.54.10-4	15	0.0118	13.97	3453	0.632
$11 \cdot 10^{-4}$	20	0.0118	13.92	3441	0.637

When the combustion chamber volume changes, the pressure, thrust, and working time of the engine also change but not significantly (volume increases by 20%, pressure decreases by 1.07%, thrust decreases by 1.07%, engine working time increases by 2.9%). The change in volume parameters does not greatly affect the engine's working parameters.

3.3 Investigate the influence of the throat cross-sectional area of the nozzle

To evaluate the influence of the throat cross-sectional area of the nozzle on the pressure and thrust of the engine, the throat cross-sectional diameter is changed and the engine combustion chamber volume is kept constant. The survey results are shown in Table 4.

Throat cross- sectional diameter parameters (m)	Change of throat cross- sectional parameters (%)	Volume of the combustion chamber (m ³)	Maximum pressure value (MPa)	Maximum thrust value (N)	Engine working time (s)
0.011918	1	9.165·10 ⁻⁴	13.35	3356	0.648
0.012036	2	9.165·10 ⁻⁴	12.60	3202	0.674
0.012154	3	9.165·10 ⁻⁴	11.90	3073	0.700
0.01239	5	9.165·10 ⁻⁴	10.64	2828	0.758

Table 4. Table of pressure, thrust, and engine's working time

The results in Table 4 show that:

The change in the critical cross-section of the nozzle has a great influence on the engine's working parameters. Throat cross-sectional diameter increased by 1%, pressure decreased by 6.03%, thrust decreased by 4.4%, and engine working time increased by 4.45%.

This change is due to the increase in the critical crosssectional area of the nozzle, causing the gas flow to increase, the pressure of combustion products in the engine's combustion chamber to decrease, and the combustion speed and gas flow also decrease, making the engine work time increase.

3.4 Investigate the effect of propellant rod length

The basic characteristics of the propellant's shape are the burning surface area *S* and burning thickness e_1 . The burning surface area determines the energy intensity and gas flow produced, The generated gas flow \dot{m}_+ is proportional to *S* according to the following expression:

$$\dot{m}_{+} = Su\rho_{T} \tag{16}$$

The ratio between the burning surface area and the critical cross-sectional area of the nozzle is a quantity that directly affects the pressure in the combustion chamber, so it also affects the value of the thrust force. The burning thickness of the propellant e_1 affects the burn-out time of the propellant, which in turn affects the engine's working time:

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$$t_k = \int_0^{e_1} \frac{de}{u} \approx \frac{e_1}{u} \tag{17}$$

To evaluate the influence of propellant geometrical parameters on engine pressure and thrust, the propellant rod length was varied while leaving the engine dimensions unchanged. The survey results are shown in Table 5.

Table 5. Value of pressure, thrust, engine's working time

Length of propellant rod (m)	Rate of change (%)	Volume of the combustion chamber (m ³)	Maximum pressure value (MPa)	Maximum thrust value (N)	Initial burning surface area of the propellant dose (m ²)	Engine working time (s)
0.3953	1	9.165 · 10 ⁻⁴	14.34	3549	0.0723	0.613
0.3998	2	9.165 · 10 ⁻⁴	14.67	3631	0.0732	0.601
0.4038	3	9.165 · 10 ⁻⁴	14.97	3698	0.0739	0.589
0.4077	4	9.165 · 10 ⁻⁴	15.24	3761	0.0746	0.579
0.4116	5	9.165 · 10 ⁻⁴	15.52	3842	0.0753	0.562

When changing the length of the propellant rod (without changing the mass of the propellant rod), the pressure, thrust, and working time of the engine change rapidly (propellant rod length increases by 5%, pressure increases by 9.77%, thrust increased by 9.75%, engine working time decreased by 8.5%).

4. CONCLUSIONS

The article presents a method for determining the working parameters of a solid-fuel rocket engine using Matlab software. The mathematical model is relatively complete, close to the requirements of the optimal design problem of solid-fuel rocket engines. The results of the article are very important in the process of designing solid-fuel rocket engines, minimizing the number of tests, thus bringing high economic value. With the survey results obtained, some conclusions are drawn as follows:

- The volume of the combustion chamber and the critical area of the nozzle significantly affect the working parameters of a solid-fuel rocket engine. Small changes in these parameters also cause significant deviations of the rocket engine's thrust.

- The burning thickness and burning speed of the propellant are parameters that significantly affect the combustion rules and thrust of the rocket engine. Changing the burning thickness is one of the methods of changing the law of thrust over time.

- In the process of designing and manufacturing solid-fuel rocket engines, the throat cross-sectional area parameters of the nozzle need to be determined accurately. It is necessary to choose high-precision machining methods and machining tools to limit errors in the structural parameters of rocket engines.

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