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Virtual Extended Beamforming for Circular Arrays Based on Particle Swarm Optimization

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Abstract: Aiming at the problems of high side lobe level, wide main lobe and high data complexity in beamforming of uniform circular array, this paper studies the beamforming method of uniform circular array. Firstly, the uniform circular array is converted into a virtual uniform linear array by using the phase mode excitation technology. On this basis, the particle swarm optimization algorithm is applied to optimize the beamforming. Finally, the equivalent array aperture is further expanded by using the linear prediction algorithm. This method not only retains the advantage of omnidirectional scanning of the circular array, but also fully utilizes the convenience of the mature algorithms of the linear array. At the same time, the resolution and freedom of the beamforming are significantly improved through aperture expansion.

Keywords: Beamforming: Uniform Circular Array: Particle Swarm Optimization: Element Aperture Expansion

1. Introduction

With the rapid development of wireless communication and radar systems, beamforming technology, as one of the core methods in array signal processing, plays a significant role in enhancing signal gain, suppressing interference, and improving target detection capabilities. Traditional beamforming methods, such as Delay-and-Sum (DAS) [1] and Minimum Variance Distortionless Response (MVDR)[2], have demonstrated good performance in many scenarios. However, their performance is often limited in complex electromagnetic environments or under strong noise interference. Therefore, exploring more efficient and robust beamforming algorithms has become one of the current research hotspots.

In recent years, Phase Mode Excitation (PME)[3] has garnered widespread attention due to its exceptional performance in Uniform Circular Array (UCA)[4]-[5]. By converting the array output into the phase mode domain[6], this method effectively simplifies the computational complexity of beamforming and enhances the symmetry and controllability of the directional pattern. However, traditional PME methods still exhibit certain limitations when dealing with non-stationary signals or dynamic interference. To address this, researchers have attempted to integrate adaptive signal processing techniques, such as Linear Prediction (LP), to further enhance the system's anti-interference capability and the flexibility of beamforming. Linear Prediction, by exploiting the temporal correlation of signals, can effectively estimate and suppress noise and interference, thereby optimizing the performance of beamforming.

Meanwhile, significant progress has been made in the application of intelligent optimization algorithms[7] in beamforming. As an efficient global optimization method, Particle Swarm Optimization (PSO)[8-10] is widely used for weight optimization in beamforming due to its fast convergence speed and flexible parameter adjustment. By combining the PSO algorithm[11]-[12], the optimal weight vector can be searched more efficiently, thus achieving better beamforming effects in complex environments.

2. Array Data Model

The uniform circular array is composed of N antenna elements uniformly distributed on the circumference, situated on the xoy plane, with each element positioned on a circle with a radius of r. As depicted in Figure 1, the signal source is located in the far field of the array, allowing the incident signal to be approximated as a plane wave. This article solely considers the scenario where the array elements are coplanar with the incident signal.

Let the position of the center of the UCA be the origin of the coordinate system, and the signal incidence angle be the counterclockwise angle between the incoming wave signal and the x-axis, denoted as $\theta \in [0, 2\pi]$; the angle between the nth array element and the x-axis is $\gamma_n = 2\pi(n-1)/N$, and its position vector is $p_n = [R\cos\gamma_n, R\sin\gamma_n]^T$.

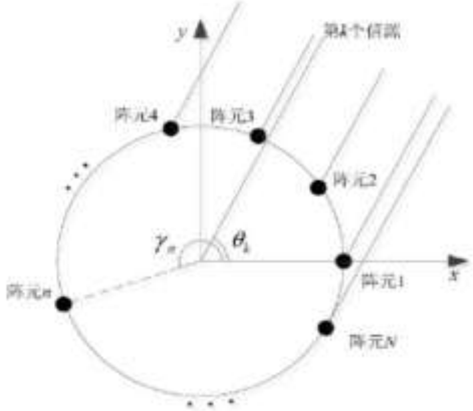

Figure 1.Circular array structure

Assuming at time t, there are K far-field narrowband plane waves reaching the array, with an incident angle denoted as, k=1,2,…,K。 Consider signal propagation from the direction of $\gamma_k = (\cos\theta_k, \sin\theta_k)$. The delay of the signal between the nth element and the center of the circle :

$$\Gamma_n = \gamma_k p_n / c = R\cos(\theta_k - \gamma_n)/c \tag{1}$$

At the same time, the phase difference between the complex envelope of the signal received by the nth element and the signal received by the center of the circle :

$$\psi_n = e^{j2\pi f_0 \tau_n} = e^{j\frac{2\pi}{\lambda}\gamma_k \tau_n} = e^{j\frac{2\pi}{\lambda}R\cos(\theta_k - \gamma_n)} \tag{2}$$

Among them, c is the speed of light, f is the center frequency of narrowband signals, and $\lambda = c / f_0$ is the wavelength of the signal.

The guiding vector of the signal:

$$X(t) = A(\theta)S(t) + N(t) \tag{3}$$

In the formula, $X(t) = [x_1(t), x_2(t), ..., x_N(t)]^T$ is the single snapshot of the received data, $A(\theta) = [a(\theta_1), a(\theta_2), ..., a(\theta_N)]^T$ is the steering vector, and $N(t) = [n_1(t), n_2(t), ..., n_N(t)]^T$ is the noise vector.

According to equation (3), the array manifold structure of UCA is relatively complex, and many mature beamforming algorithms are difficult to apply. The array manifold of UCA is relatively complex, and the array radius R and the number of elements N will affect the beamforming effect.

Considering a fixed signal-to-noise ratio of 20db and a fixed element radius of 0.5, beamforming was performed using conventional beamforming methods. The results are shown in Figure 2

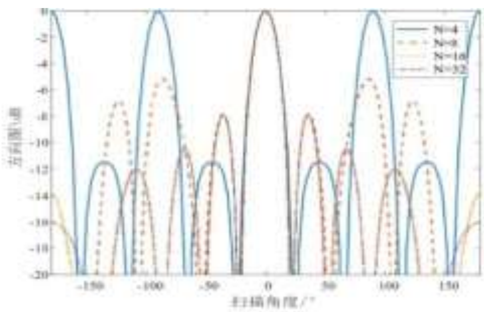

Figure 2.The influence of the number of array elements on the beamforming results

According to the results shown in Figure 2, when the array radius is fixed, the main lobe width of the beam is not affected by the number of elements, but the sidelobe level is significantly affected by the number of elements. When the number of array elements is too small, the sidelobe level will rise, seriously affecting the performance of the beam.

Still considering a signal-to-noise ratio of 20db, with a fixed number of 16 elements, the element radii are changed to 0.5 λ , 1.5 λ , and 2 λ in sequence. The result of beamforming is shown in Figure 3.

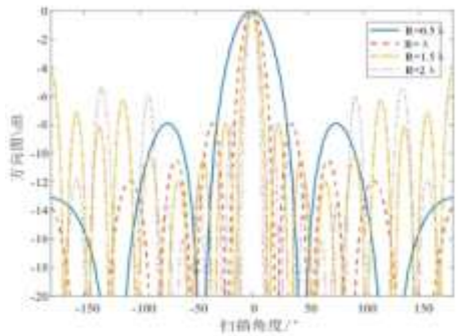

Figure 3.The influence of element radius on beamforming results

As can be seen from Figure 3, with the increase in array radius, the main lobe width of the beam gradually decreases, but the sidelobe level rises. The increase in the sidelobe level is due to the increase in the spacing between array elements.

In summary, the pattern of a UCA is determined by both the array radius and the number of array elements. Therefore, improving the beam quality cannot be achieved by simply altering a single part. To address this issue, this paper proposes a hybrid beamforming method based on phase mode excitation, linear prediction, and particle swarm optimization algorithm, aiming to enhance the performance of the array system in dynamic interference environments. Firstly, phase mode excitation is utilized to convert the array output to the mode domain, reducing computational complexity. Secondly, linear prediction technology is combined to perform adaptive suppression of interference signals. Finally, the particle swarm optimization algorithm is employed to optimize the weights of beamforming, achieving higher signal gain and lower sidelobe levels. Simulation and experimental results demonstrate that the proposed method outperforms traditional methods in terms of interference suppression, beam pointing accuracy, and computational efficiency, providing a new solution for beamforming in complex environments.

3. Phase mode excitation

For ULA, increasing the number of array elements can enhance the effect of beamforming. UCA does not possess a Vandermonde structure, making it extremely difficult and unlikely to expand the array elements of UCA. Therefore, drawing on methods from the literature, we adopt phase mode excitation to convert the data format of UCA into that of ULA, and then proceed with array element expansion.

The phase mode excitation of UCA is actually the processing of the excitation function for a continuous circular aperture through Fourier transform. When the signal comes from , and a continuous circular aperture with radius R is excited, the resulting normalized far-field pattern :

$$f(\theta) = \frac{1}{2\pi}\int_0^{2\pi} \omega(\gamma)\exp\left(j\frac{2\pi R}{\lambda}COS(\theta - r)\right)dr \tag{4}$$

While adding a transformation matrix:

$$T = (1/N)J^{-1}F_e \tag{5}$$

In the formula:

$$F_e = [w_{-L,} w_{-L+1}, ..., w_L]^H \tag{6}$$

$$w_L = [1, e^{j\frac{2\pi l}{N}}, \ldots, e^{j\frac{2\pi l(N-1)}{N}}]^H \tag{7}$$

$$J = diag[i^l J_l(\beta)] \tag{8}$$

Where L represents the element number of the virtual uniform linear array.The number of elements after phase mode excitation is 2L+1.

When the number of array elements N of the UCA is greater than 2L+1, the received data X(t) of the UCA is transformed using a transformation matrix T to obtain

$$\tilde{X}(t) = TX(t) = \tilde{A}(\theta)S(t) + \tilde{N}(t) \tag{9}$$

In the formula:

$$\tilde{X}(t) = [x_{-L}, x_{-L+1}, \ldots x_L]^T \tag{10}$$

$$\tilde{A}(\theta) = [\tilde{a}(\theta_1), a(\theta_2), \ldots, a(\theta_K)]^T \tag{11}$$

$$a(\theta_K) = [e^{-iL\theta_K}, e^{-i(L-1)\theta_K}, \ldots, e^{iL\theta_K}]^T \tag{12}$$

The steering vector of the virtual linear array excited by phase mode is similar to that of the ULA. Its phase varies linearly with the array element.

Phase mode excitation technology brings core advantages to uniform circular array processing. Its greatest value lies in the mathematical transformation that converts a physical circular array into an equivalent virtual uniform linear array in the beam domain. This transformation brings revolutionary benefits: it enables many mature, efficient, and computationally low-complexity classical high-resolution algorithms to be directly transplanted and applied to circular arrays. This is because the array manifold Vandermonde structure, upon which these algorithms rely, is perfectly presented in the phase mode output of the virtual linear array, thus avoiding the immense difficulty of redeveloping complex algorithms for circular arrays.

4. Particle swarm optimization algorithm and linear prediction

4.1 Particle swarm optimization algorithm

Consider a virtual linear array that has undergone phase pattern excitation. Assuming the central array element as the reference element, 2L+1 array elements are arranged on both sides. When the array simultaneously receives K signals from the far field at a certain moment, the single snapshot of the l-th array element in the virtual linear array can be expressed as:

$$x_1(t) = \sum_{k=1}^{K} s_k(t) \times e^{jl\theta_k} + n_1(t) \tag{13}$$

The received signal of a virtual linear array can be regarded as consisting of three parts: the incident signal $s_k(t)$, the delay factor $e^{jl\theta_k}$, and noise $n_1(t)$. If estimates of these three parts are obtained through certain methods based on known data, then the data received by the extended array elements can be constructed, thereby expanding the array aperture size

Define the cost function as:

$$C = \sum_{l=-L}^{L} | x_1(t) - \sum_{k=1}^{K} s_k(t) \times e^{jl\theta_k} + n_1(t) |^2 \tag{14}$$

The PSO algorithm simulates the foraging behavior of bird flocks, achieving global optimization through the collaboration between individual birds and the flock. Based on equation (14), a 3K-dimensional solution space is constructed, within which Q particles are randomly arranged. After the m-th optimization step, the position and velocity of the q-th particle are respectively:

$$\rho_q(m) = (p_1(m), p_2(m), \ldots, p_{3K}(m)) \tag{15}$$

$$v_q(m) = (v_1(m), v_2(m), \ldots, v_{3K}(m)) \tag{16}$$

After optimizing the m-th step, each particle searches for a better solution nearby and updates its own historical best solution as:

$$P_q(m) = [p_1(m), p_2(m), \ldots, p_{3K}(m)] \tag{17}$$

Meanwhile, by comparing the best points obtained by all particles, the global optimum at this time is obtained:

$$g_{best}(m) = (g_1(m), p_2(m), \ldots, p_{3k}(m)) \tag{18}$$

After calculating these two values, each particle in the particle swarm updates its velocity and position according to equations (19) and (20):

$$\begin{aligned} v_q(m+1) &= w_m v_q(m) \\ &+ c_1(m) r_1(m)[P_q(m) - \rho_q(m)] \\ &+ c_2(m) r_2(m)[g_q(m) - \rho_q(m)] \end{aligned} \tag{19}$$

$$\rho_q(m+1) = \rho_q(m) + v_q(m+1) \tag{20}$$

In the formula, $w(m)$ represents the inertia weight coefficient, c_1 and c_2 are learning factors, and r_1 and r_2 are random factors, typically within the range of [0,1].

When the change in the cost function value of the population optimum between two adjacent steps is less than the preset allowable range, or when the maximum number of optimization iterations is reached, terminate particle update and end the optimization.

4.2 linear prediction algorithm

From the previous section, it can be known that after being excited by phase mode, the array structure of UCA is as shown in Figure 4

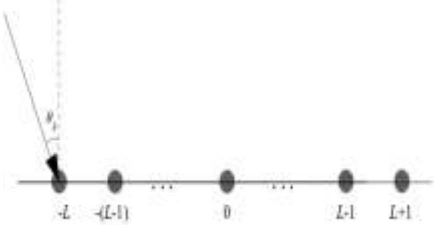

Figure 4.Virual element array structure

Linear prediction[13-15] involves estimating past and future time series based on known time series. It is achieved through a prediction filter and a prediction error filter. The prediction filter predicts the values of the desired time series, while the prediction error filter adjusts the weights of the prediction filter based on the error between the actual values and the predicted values.

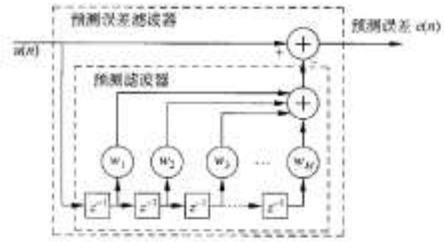

Figure 5.Schematic diagram of linear prediction

Since there is a one-to-one correspondence between the samples in the time domain and the received data from the array elements in space, for a virtual linear array with 2L+1 array elements, the snapshot data from the first 2L array elements can be used to perform forward prediction on the received data from the array element with index L. The snapshot data from the last 2L array elements can be used to perform backward prediction on the received data from the array element with index -L. Therefore, for any sampling time, there exists:

$$x_L(t)=[x_{-L}(t)\ \ x_{-(L-1)}(t)\ \dots\ x_{L-1}(t)]\begin{bmatrix} w_1^f \\ w_2^f \\ \dots \\ w_{2L}^f \end{bmatrix} \quad (21)$$

$$x_{-L}(t)=[x_{-(L-1)}(t)\ \ x_{-(L-2)}(t)\ \ \dots\ \ x_L(t)]\begin{bmatrix} w_1^b \\ w_2^b \\ \dots \\ w_{2L}^b \end{bmatrix} \quad (22)$$

In the formula, w_k^f represents the forward prediction coefficient, w_k^b represents the backward prediction coefficient, and k=1,2,…,N-1.

Similarly, using the linear prediction algorithm, we can continue to perform forward prediction to predict the received data for array elements L+1, L+2, ..., L+H. The received data for subsequent predicted array elements are added to the initial array:

$$\tilde{X}_p(t)=[\tilde{X}(t), x_{L+1}(t), \dots, x_{L+H}(t)]^T \quad (23)$$

Utilizing the expanded data, adaptive beamforming can be performed. By mining the known data and then constructing the data for the expanded array elements, beam sharpening for UCA is achieved.

5. Experiment and Simulation

This section will conduct simulation experiments on the beamforming algorithm proposed in this paper. Firstly, the effect of the linear prediction algorithm on extending the array aperture will be verified. A uniform circular array (UCA) with 16 array elements is set up, with incident angles set at 0 and 10°, and a signal-to-noise ratio (SNR) of 20dB. If the number of phase mode excitations L is set to 6, the UCA undergoes phase mode transformation to obtain a virtual uniform linear array with 13 array elements. Then, the linear prediction (LP) algorithm is used for virtual expansion. Conventional beamforming algorithms are employed to perform adaptive beamforming on both the UCA and the virtual linear array after array element expansion. The results obtained are shown in Figure 6.

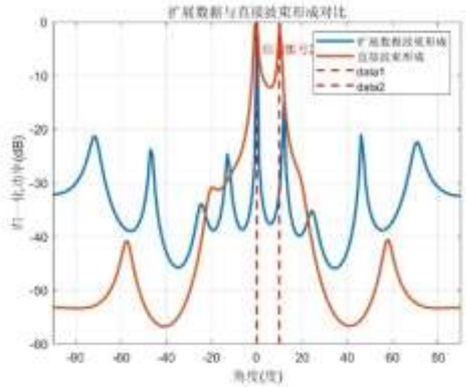

Figure 6.Analysis of the effect of linear prediction on beamforming

As can be seen from Figure 6, after phase mode excitation and linear prediction, the beamforming pattern exhibits a notably sharper main lobe, albeit with slightly elevated sidelobes. This is attributed to the limitations of phase mode excitation. Essentially, phase mode excitation involves non-uniform phase weighting. While it can compress the main lobe, it disrupts the spectral symmetry or orthogonality of the original signal, leading to spectral leakage. The leaked energy is transferred to the sidelobes, resulting in their elevation.

Therefore, this paper employs the particle swarm optimization algorithm to optimize the virtual linear array after phase mode excitation, obtaining estimates of the incident signal $s_k(t)$, delay factor $e^{jl\theta_k}$, and noise $n_1(t)$. Subsequently, linear prediction is performed to further optimize the beamforming effect. The simulation results are shown in Figure 7.

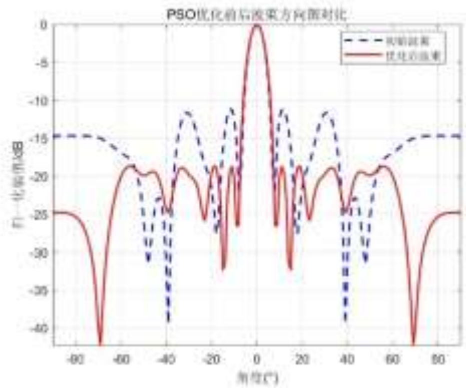

Figure 7.Comparison of beam patterns before and after optimization using PSO algorithm

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As can be seen from Figure 7, compared to the virtual linear array after element expansion without the particle swarm optimization algorithm, the sidelobes of the beamforming pattern are significantly reduced after particle swarm optimization.

6. Conclusion

To address the issues of high sidelobes, wide mainlobes, and complex data structures in beamforming for Uniform Circular Array (UCA), this paper proposes a method based on the Particle Swarm Optimization (PSO) algorithm to expand the array elements of the virtual uniform linear array corresponding to UCA[16-17]. Experimental results show that the proposed method, through effective mining of received data, enables the formation of beams with a narrower mainlobe and lower sidelobe levels using the expanded array.

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