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DOA Estimation Based on Compressed Sensing Theory

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Abstract: Array signal processing technology has been widely applied in research fields such as radar, sonar, and positioning due to its advantages of extensive spatial coverage, strong anti-interference capability, high signal gain, and superior signal resolution. Traditional array signal processing techniques are based on the Nyquist sampling theorem, which poses challenges in practical applications, such as high computational load and difficulties in achieving real-time performance. Compressive sensing (CS) theory, also known as compressed sampling principle, reconstructs the original signal accurately or approximately with only a small number of samples by leveraging the sparsity and compressibility of signals. This paper combines CS theory with the Orthogonal Matching Pursuit (OMP) algorithm, namely the CS-OMP algorithm, and applies it to Direction of Arrival (DOA) estimation, thereby realizing DOA estimation based on compressive sensing theory. Additionally, the performance of the traditional DOA estimation algorithm, the Multiple Signal Classification (MUSIC) algorithm, is compared under different scenarios, and the impact of parameters such as the number of array elements, snapshot count, and signal-to-noise ratio on the performance of the CS-OMP algorithm is further explored. This provides a novel and effective approach for complex signal processing.

Keywords: compressed sensing; sparse representation; MUSIC algorithm; compressed sampling; DOA estimation

1. INTRODUCTION

In the analysis and processing of modern signal systems, array signal processing, as an important component, has been studied by numerous scholars for decades, leading to rapid development. Its application scope includes multiple fields such as radar, sonar, wireless communication, etc[1]. The main research directions include beamforming and spatial spectrum estimation. Spatial spectrum estimation technology is the estimation of spatial signal parameters. The spatial spectrum reflects the spatial energy distribution of a signal. If you want to obtain a spatial spectrum, you need to first estimate the direction of arrival (DOA) of the signal [2], and estimate the position of the signal source by measuring the DOA or AOA of the signal source.

Nowadays, the technology of using narrowband array signal processing to achieve DOA estimation has been widely applied. The DOA estimation of narrowband signals is the foundation of array signal DOA estimation. Early narrowband signal DOA estimation methods such as conventional beamforming (CBF) [3] and minimum variance method (MVM) [4] can achieve rough estimation of the direction of incoming signals, but due to the Rayleigh limit of array aperture [5], their estimation accuracy and signal identification ability gradually cannot meet people's needs. There are two main types of traditional narrowband subspace based broadband signal DOA estimation algorithms: incoherent subspace processing (ISM) [6] and coherent subspace processing (CSM) [7]. However, the DOA estimation achieved by these two methods is only an extension from the time domain to the frequency domain, and therefore still faces the problem of poor processing ability for strongly correlated wideband signals and a decrease in DOA estimation accuracy due to a small number of sampling points in the frequency domain sub region.

The traditional spatial spectrum estimation algorithm, MUSIC algorithm, has advantages such as high resolution, high accuracy, ability to overcome Rayleigh limit limitations, and super-resolution. However, based on the traditional sampling theorem, it inevitably has significant limitations, such as large computational complexity and poor real-time performance. With the continuous increase of information volume, the requirements for signal acquisition rate and processing speed are also increasing. Therefore, this paper combines CS theory with the classic sparse signal reconstruction method OMP algorithm (CS-OMP) to accurately estimate the direction of signal arrival with fewer sampling points. This not only reduces the number of sampling points and computation, reduces storage space, but also lowers the hardware requirements, which has good research significance and implementation value.the focus of this paper is to combine CS theory with the classic sparse signal reconstruction method OMP algorithm (CS-OMP) to accurately estimate the direction of the signal wave with fewer sampling points. This not only reduces the number of sampling points and computation, but also reduces the storage space and hardware requirements. It can also bring the A/D converter to the front end of the system and compare it with the MUSIC algorithm, which has good research significance and implementation value.

2. ALGORITHM PRINCIPLE

2.1 Introduction to Traditional MUSIC Algorithm

Assuming a uniform linear array consisting of M units, with unit spacing d and number of sources p, and input signal $s_i = (\theta)$ from the mth unit, the output vector of the array is as follows:

$$x(t) = [x_0(t), x_1(t), ..., x_{M-1}(t)]^T \qquad (2\text{-}1)$$

Based on the first element, the sound path difference of signal $s_i(t)$ from the mth element to the reference point is $\Delta_m = (m-1)d\sin(\theta_i)$, and the corresponding time delay is $\tau_{mi} = [(m-1)d\sin(\theta_i)]/c$. The output of the array element can be expressed as:

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$$x_m(t)=\sum_{i=1}^{p}s_i(t)e^{-jw\tau_{mi}}+n_m(t) \quad (2\text{-}2)$$

In the equation, $n_m(t)$ is the noise on the mth element, $\omega=2\pi f_0$, then the output vector of the array can be expressed as:

$$X(t)=As(t)+n(t) \qquad (2\text{-}3)$$

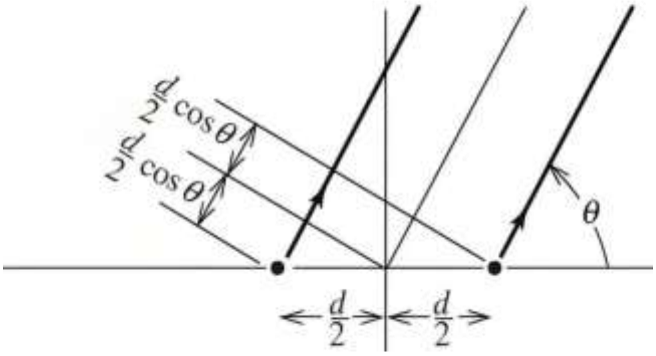

Figure. 1 Array signal receiving model

Among which

$A(\theta)=[a(\theta_1),a(\theta_2),...,a(\theta_p)]$, is the array direction matrix of $M\times P$;

$a(\theta_1)=[1,e_{1i}^{-jw\tau},...,e_{(m-1)i}^{-jw\tau}]^T$, is the direction vector of the signal source $s_i=(t)$;

$s(t)=[s_1(t),s_2(t),...,s_p(t)]^T$, is the signal vector of $P\times 1$;

$n(t)=[n_0(t),n_1(t),...,n_{M-1}(t)]^T$, is an additive noise vector of $M-1$.

Assuming that each element of the array satisfies the space-time white noise condition, the autocorrelation matrix of the array output vector is defined as:

$$R=E[x(t)x^H(t)]=AR_sA^H+\sigma^2 I \quad (2\text{-}4)$$

Among them, $R_s=E[s(t)s^H(t)]$ is the autocorrelation matrix of the signal.

Perform eigenvalue decomposition on R to achieve high-resolution target orientation estimation based on the orthogonality of signal subspace and noise. At the same time, it is easy to prove $R=R_H$, indicating that the array covariance matrix belongs to the Hermitian matrix. The eigenvalues are positive, which is $\lambda_i(i=1,2,...,M)$, and the corresponding eigenvector is $\mu_i(i=1,2,...,M)$. Therefore, the eigenvalue decomposition of the covariance matrix is [8]

$$R=U\Lambda U^H=\sum_{i=1}^{M}\lambda_i\mu_i\mu_i^H \qquad (2\text{-}5)$$

In the formula, $U=[\mu_1,\mu_2,...,\mu_M]$ is a unitary matrix composed of eigenvectors, and $\Lambda=diag[\lambda_1,\lambda_2,...,\lambda_M]$ is a diagonal matrix composed of eigenvalues.

Sort the eigenvalues of R in descending order, and the eigenvectors corresponding to the top P largest eigenvalues form a unitary matrix $U_s=[\mu_1,\mu_2,...,\mu_p]$, which is stretched into a signal subspace. The eigenvectors corresponding to the last M-P minimum eigenvalues form a unitary matrix $U_n=[\mu_{p+1},\mu_{p+2},...,\mu_M]$, which expands into a noise subspace. Assuming that the signal correlation matrix $R_s=E[s(t)s^H(t)]$ is non singular, meaning that the signals are uncorrelated, it can be proven that the array direction matrix A and the subspace formed by the signal subspace are the same, and because $U=[U_s,U_n]$ is a unitary matrix, $U_s^HU_n=0$. The spatial spectrum of MUSIC algorithm can be defined as:

$$P_{MUSIC}(\theta)=\frac{1}{a^H(\theta)U_nU_n^Ha(\theta)} \qquad (2\text{-}6)$$

The spectral peak search is used to obtain the estimated value $\hat{\theta}_i, i=1,2,...,p$ of the direction of arrival for the spatial spectrum mentioned above. However, in reality, R is unknown, but it can be estimated based on the input data x (t) from the array of W snapshots. The estimation formula is

$$\hat{R}=\frac{1}{N}\sum_{i=1}^{N}x(t)x^H(t) \qquad (2\text{-}7)$$

Perform eigenvalue decomposition on $\hat{R}$ to obtain noise subspace estimation, which can then obtain MUSIC spatial spectrum and direction of arrival estimation. The basic idea of MUSIC algorithm is to divide the observation space into two parts, one is the noise subspace with only noise, and the other is the signal subspace composed of noise and signal. Based on the orthogonality of these two spaces, a cost function is constructed to perform DOA estimation [9].

2.2 Basic principles of compressive sensing

The compressive sensing theory states that if a signal satisfies sparsity or compressibility, it can be sampled at a sampling frequency much lower than Nyquist, and accurate reconstruction of the original signal can be achieved. The core of this theory consists of the following three parts:

1) Sparse representation of the original signal;

2) Construct appropriate measurement matrices for different situations;

3) Sparse reconstruction of the above signal.

Assuming signal x is a discrete signal with a length of N, it can be represented by a set of orthogonal bases:

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$$x = \sum_{i=1}^{N} \psi_i \alpha_i = \Psi \alpha \qquad (2\text{-}8)$$

In the formula: $\psi = [\psi_1, \psi_2, ..., \psi_N]$ represents the standard orthogonal basis, $\alpha = [\alpha_1, \alpha_2, ..., \alpha_N]^T$ represents the coefficient vector. For the weight coefficient vector α ,It can be defined as:

$$\alpha = \Psi^T x \qquad (2\text{-}9)$$

If 0<P<2 and 0<k, these coefficients satisfy

$$\| \alpha \|_p = (\sum_{i=1}^{N} | \alpha_i |^p)^{\frac{1}{p}} \le k \qquad (2\text{-}10)$$

In the formula, where $\| \alpha \|_p$ represents the p-norm of vector α , it indicates that the weight coefficient vector α can be sparsely represented in a certain transformation domain, and also indicates that the signal has compressibility.

Under the premise that signal x can be sparsely represented or compressed, different observation matrices can be used to project and transform the sampled signal, which can reduce the dimensionality of the original high-dimensional signal and obtain the desired sparse signal. The observation vector is:

$$y = \Phi x = \Phi \psi \alpha = \Theta \alpha \qquad (2\text{-}11)$$

In the equation: $\Theta = \Phi \psi$, where Φ is the $Q \times N$ -dimensional measurement matrix, $N >> Q$ and $Q \ge O(K \lg(N/K))$ sparse sampling of the signal can be achieved through equation (2-11)

For the construction of the compressive sensing matrix Θ , reference [10] proves through mathematical formula derivation that Θ can be solved using algorithms such as the minimum l_0 norm method and orthogonal matching tracking when satisfying the restricted isometric principle (RIP) criterion, thus achieving high-precision DOA estimation based on compressive sensing.

2.3 Sparse representation

In this article, we consider a discrete signal x of length N, denoted as $x(n)$, $n = [1, 2, ..., N]$. According to signal theory, any N-dimensional vector can be represented by a linear combination of a set of N × 1-dimensional basis vectors $\{\Psi_i\}_{i=1}^{N}$ [11]. Signal x can be expressed as:

$$x = \sum_{i=1}^{N} s_i \psi_i \quad \text{or} \quad x = \Psi S \qquad (2\text{-}12)$$

In the equation, $s_i = \langle x, \psi_i \rangle$, S and x are N×1-dimensional column vectors, $\Psi = [\psi_1, \psi_2, ..., \psi_N]$ is an N × N-dimensional basis matrix, and the sparse representation of the signal is that when x is projected onto the orthogonal transformation basis Ψ , the absolute values of the majority of the transformation coefficients are very small, resulting in a sparse or approximately sparse transformation vector $S = \Psi^T x$. When there are only K (K<<N) non-zero coefficients in S , and the other N-K coefficients are all 0 (or very small), they can be expressed as $K = \| S \|_{l_0}$ in the l_0 norm sense. Therefore, the signal x can be considered K-sparse, and the corresponding Ψ is called a sparse basis. This K-term sparse representation can be used to approximate the original signal well.

2.4 Measurement code

In this model, the original signal x is projected onto another M × N-dimensional observation matrix Ψ that is not related to the transformation basis $\Phi = [\varphi_1, \varphi_2, ..., \varphi_m, ..., \varphi_M]$, resulting in an M × 1-dimensional matrix y. At this point, M<<N, the original signal x is compressed and sampled. The expression is

$$y = \Phi x \qquad (2\text{-}13)$$

Substituting equation (2-12) into equation (2-13) yields

$$y = \Phi x = \Phi \Psi S = \Theta S \qquad (2\text{-}14)$$

In the equation, Θ is an M × N-dimensional matrix.

Because M<<N, the above matrix equation has no definite solution and is an underdetermined equation. There are countless S that satisfy the condition, but S is sparse. As long as a suitable observation matrix Φ is designed, the above equation can be guaranteed to have a unique sparser solution $\hat{S}$.

At this point, matrices Θ and Ψ should be as orthogonal as possible, and their coherence is defined as：

$$\mu(\Phi, \Psi) = \sqrt{N} \times \max_{1 \le k, j \le N} | \langle \varphi_k \psi_j \rangle | \qquad (2\text{-}15)$$

The above equation μ should be as small as possible, that is, the observation matrix and sparse basis should be as uncorrelated as possible.

To accurately recover the original signal, the matrix Θ must satisfy the RIP criterion, which defines the equidistant constant δ_K of the matrix Θ for any transformation vector S with strict K-sparsity, and where δ_K is the minimum value satisfying the following equation

$$(1 - \delta_K) \| S \|_2^2 \le \| \Theta S \|_2^2 \le (1 + \delta_K) \| S \|_2^2 \qquad (2\text{-}16)$$

When $\delta_K < 1$, the matrix Θ satisfies K-order RIP, and when $\delta_{2K} + \delta_{3K} < 1$, distortion free recovery can be achieved, and Φ and Ψ are uncorrelated, which is an equivalent case of the RIP criterion mentioned above.

2.5 Signal reconstruction

When the matrix Θ satisfies the RIP criterion, the sparser solution $\hat{S}$ can be obtained from the observed values y first, and then the original signal can be recovered based on equation

$\hat{x} = \hat{\Psi}\hat{S}$. We can consider the above process as an optimization problem, as follows:

$$\min \| S \|_{l_c} \text{, s.t. } y = \Theta S \quad (2\text{-}17)$$

For the above equation, the norm mentioned above can only represent sparsity well when it is $0 \le c \le 1$. Therefore, there are

$$\min_{s} \| S \|_{l_0} \text{, s.t. } y = \Theta S \quad (2\text{-}18)$$

But solving the l_0 norm problem is an NP problem, and in this case, the l_1 norm problem and the l_0 norm problem are equivalent. Solving the optimization problem will produce the same solution, so the above equation becomes

$$\min_{s} \| S \|_{l_1} \text{, s.t. } y = \Theta S \quad (2\text{-}19)$$

The solution is

$$\hat{S} = \operatorname*{argmin}_{s} \| S \|_{l_1} \text{, s.t. } y = \Theta S \quad (2\text{-}20)$$

The computational complexity of the above equation is $o(N^3)$, therefore the sparse expression of $\hat{S}$ on the basis of Ψ can be simplified into linear programming.

On the basis of the above, it can be converted into the following equation and then solved through second-order cone programming.

$$\min_{s} \| S \|_{l_1} \text{, s.t. } \| \Phi x - y \|_2 \le \varepsilon \quad (2\text{-}21)$$

3. SIMULATION RESULT ANALYSIS

3.1 Experiment 1: Comparison of effectiveness between MUSIC algorithm and CS algorithm

The purpose of this experiment is to compare and analyze the CS algorithm and MUSIC algorithm to explore which of the conventional MUSIC algorithm can achieve better estimation results for DOA estimation using compressive sensing principle under two prior conditions of known and unknown number of sources. This experiment uses two sources with an incident angle of [-20° 40 °], 12 array elements, a signal-to-noise ratio of 20 dB, and 1000 snapshots.

When we separately tell the MUSIC algorithm that the number of sources is 1, 2, and 9, we obtain the following results:

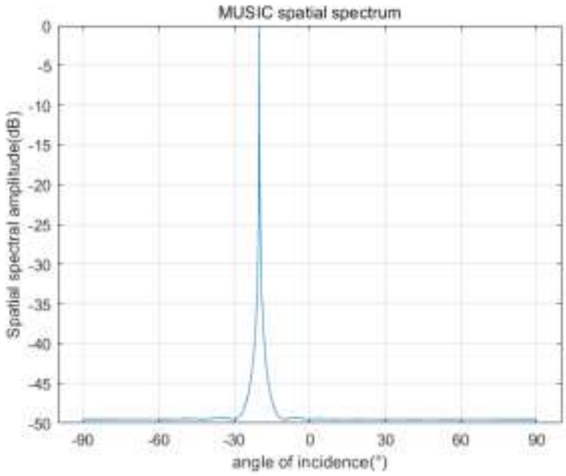

Figure. 2 MUSIC algorithm solution result (indicating that the number of signal sources is 1)

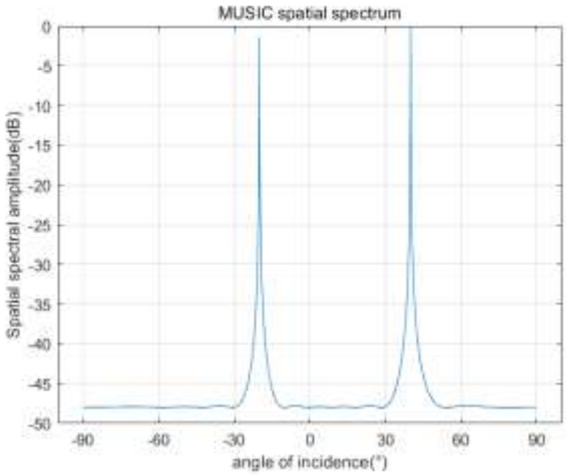

Figure. 3 MUSIC algorithm solution result (indicating that the number of signal sources is 2)

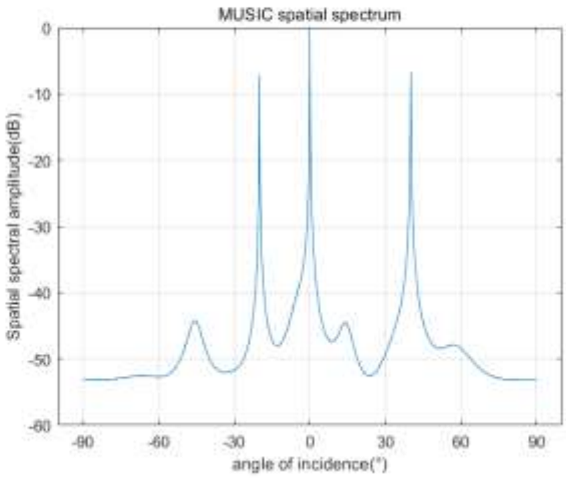

Figure. 4 MUSIC algorithm solution result (indicating that the number of signal sources is 9)

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When the number of radiation sources notified matches the actual number of radiation sources, the simulation results in Figure 3 are obtained, and the MUSIC algorithm accurately achieves DOA estimation. But when the number of radiation sources was modified to 1 or 9, simulation results were obtained as shown in Figures 2 and 4, respectively, with estimated values showing missing or deviated values and false peaks. This is because the MUSIC algorithm performs eigenvalue decomposition on the received signal during the reconstruction process. The algorithm defaults to the number of input radiation sources as the number of eigenvalues, resulting in a deviation in the division of the signal subspace and noise subspace, leading to inaccurate DOA estimation values.

Figure 5 shows the solution results of the CS algorithm. As the compressive sensing algorithm is based on the premise that the signal has sparsity or compressibility, it captures the characteristic relationship of the signal through the observation matrix. Therefore, CS does not need to inform the number of sources in advance. As long as the signal itself has sparsity, it can find the sparse representation of the signal through optimization algorithms, thereby achieving accurate estimation of signal parameters.

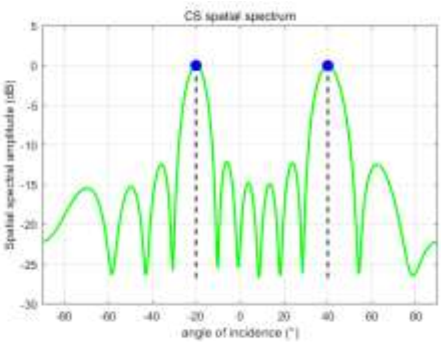

Figure. 5 CS algorithm solution result

In practical applications, due to interference from noise and other factors, the exact number of sources cannot be known in the vast majority of cases, especially when the source target is in a moving state. The difficulty of MUSIC algorithm estimation also increases. Through experimental comparison, it can be seen that under the prior condition of unknown number of sources, CS algorithm has better adaptability compared to MUSIC algorithm.

3.2 Experiment 2: Measuring estimation performance through root mean square error

In DOA estimation experiments, there are many methods to measure estimation performance, and the most important indicator is the size of the root mean square error. This definition is：

$$RMSE = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(\hat{x}_i(m) - x_i)^2}$$

In the equation, $\hat{x}_i(m)$ represents the incident angle obtained from the mth Monte Carlo simulation experiment.

1) Comparative experiment on the root mean square error of a single signal source as a function of signal-to-noise ratio. Assuming the position of the single signal source is 40 °, the simulation experiment is set to 500 snapshots and kept constant. The signal-to-noise ratio varies from -10dB to 30 dB at intervals of 5 dB. The mean square root error curves of the two algorithms are compared when the signal-to-noise ratio changes. To ensure the accuracy of the experiment, 200 Monte Carlo experiments are conducted at each signal-to-noise ratio, and the results are shown in Figure 6

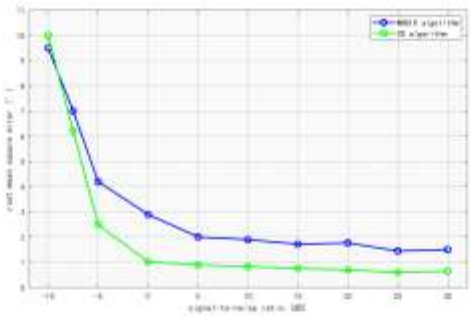

Figure. 6 RMS error curves of two algorithms for a single source with varying signal-to-noise ratio

As the signal-to-noise ratio increases, the root mean square error of both algorithms decreases to varying degrees. In the case of a single source, the estimation accuracy of the traditional MUSIC algorithm mentioned above also has a certain gap compared to the algorithm proposed in this paper. This is because the algorithm proposed in this paper uses compressive sensing theory to obtain more signal information, improves the utilization of array signals, and thus enhances the estimation accuracy.

2) Compare the root mean square error curves of two algorithms with a constant signal-to-noise ratio of 10 dB and varying snapshot counts from 20 to 200 at intervals of 20. To ensure the accuracy of the experiment, 200 Monte Carlo experiments were conducted for each snapshot count, and the experimental results are shown in Figure 7.

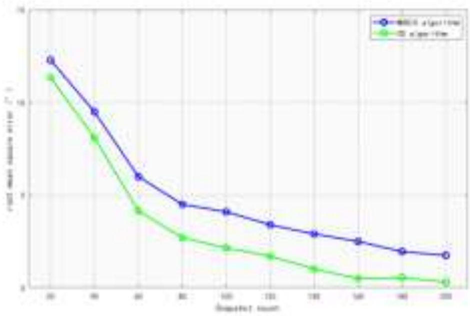

Figure. 7 Comparison of Root Mean Square Error of Two Algorithms as a Function of Fast Shot Count

From Figure 7, it can be seen that both algorithms have a decreasing trend in root mean square error as the number of snapshots increases. When the number of snapshots is the same, the root mean square error of our algorithm is lower than that of the MUSIC algorithm, indicating that our algorithm has better performance.

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3.3 Experiment 3: Comparison of DOA Estimation Success Rates between Two Algorithms

Firstly, the definition of successful experiment is: if the estimated angle differs from the true angle within ± 2 °, it can be considered as a successful estimation. Assuming the incident angles are -50 °, 10 °, and 70 °, respectively.

1)Compare the success rate changes of two different algorithms with varying signal-to-noise ratios. In the experiment, the number of snapshots was set to 5 and kept constant, and the signal-to-noise ratio varied from -10dB to 10dB at 2dB intervals. To ensure the accuracy of the experiment, 200 Monte Carlo experiments were conducted for each snapshot, and the experimental results are shown in Figure 8.

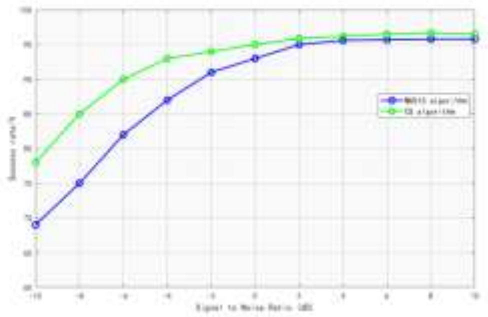

Figure. 8 Comparison of Success Rates of Two Algorithms with Changes in Signal to Noise Ratio

2)Compare the success rates of three algorithms as the number of snapshots changes. In the experiment, the signal-to-noise ratio was set to 10 dB and kept constant. The number of snapshots varied from 20 to 200 at intervals of 20. To ensure the accuracy of the experiment, 200 Monte Carlo experiments were conducted for each snapshot, and the experimental results are shown in Figure9.

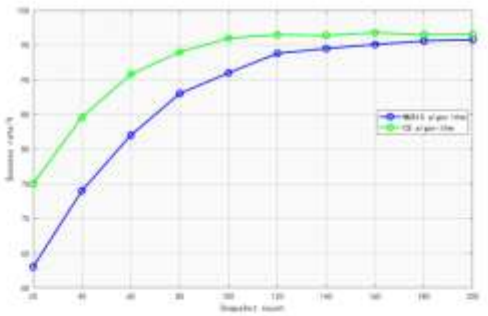

Figure. 9 Comparison of success rates of two algorithms with changes in snapshot count

From the comparison of the success rates of the two algorithms with changes in signal-to-noise ratio and number of snapshots in Figures 8 and 9, it can be seen intuitively that as the signal-to-noise ratio or number of snapshots increases, the success rates of both algorithms improve. However, under the same conditions of number of snapshots or signal-to-noise ratio, the performance of our algorithm is better than that of traditional algorithms.

4. CONCLUSION

The DOA estimation method based on compressive sensing theory proposed in this article combines compressive sensing theory with DOA estimation. And the feasibility of the algorithm was verified through MATLAB simulation experiments. At the same time, experiments were designed to compare the proposed algorithm with the traditional MUSIC algorithm under multi snapshot and low signal-to-noise ratio conditions. Compared with the traditional algorithm, the proposed algorithm has the following advantages:

1)By using compressive sensing, the elevation and azimuth angles of a signal can be accurately restored with a small amount of signal, which can effectively reduce computational complexity.

2)It is more practical to capture the characteristic relationship of signals by constructing observation matrices without the prior condition of the number of signal sources.

3)The compressed sensing algorithm has better recognition success rate and robustness compared to the traditional MUSIC algorithm.

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