Application of Fuzzy Topsis in Analyzing and Solving Transportation System Problems

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Abstract: This study addresses transportation-related problems encountered in industrial systems and proposes a decision-making methodology based on the Fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method. The aim is to select the most suitable shuttle system among multiple alternatives by evaluating various qualitative and quantitative criteria under uncertainty. Key evaluation criteria include cost, operational convenience, flexibility, reliability, safety, and after-sales service. Due to the vagueness and subjectivity inherent in expert judgments, linguistic variables are converted into triangular fuzzy numbers to construct the decision matrix. The method involves calculating the weighted normalized fuzzy decision matrix, determining fuzzy positive and negative ideal solutions, and computing the closeness coefficients for each alternative. The final ranking reveals that Alternative A2 demonstrates the highest similarity to the ideal solution and is therefore identified as the most appropriate shuttle system. The study highlights the effectiveness of the Fuzzy TOPSIS method in handling complex, multi-criteria decision-making problems in uncertain environments.

Keywords: Fuzzy TOPSIS, Transportation Systems, Fuzzy Logic, Decision Analysis, Uncertainty Modeling

1. INTRODUCTION

Transportation plays a crucial role in the operational efficiency of industrial systems. The selection of transportation systems involves considering multiple factors, making it a complex decision-making process. Traditional decision-making methods may not provide accurate results in such multi-criteria and uncertain environments. In this context, advanced techniques like the Fuzzy TOPSIS method offer a way to minimize the effects of uncertainty and subjective judgments, leading to more reliable and precise outcomes. This study examines how the Fuzzy TOPSIS method can be applied to the selection of transportation systems and how the results obtained can be beneficial in industrial systems.

2. FUZZY TOPSIS METHOD

The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method is a technique that can be used in multi-criteria decision-making problems, which can be defined as “m” alternatives in an “n”-dimensional space.

The fundamental principle of the method is that the selected alternative should be closest to the positive ideal solution and farthest from the negative ideal solution. In this context, TOPSIS can be described as an index measuring the similarity to the positive ideal solution and the distance from the negative ideal solution.

2.1 Literature

Hwang and Yoon (1981) developed the TOPSIS method based on the idea that the optimal solution should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. This idea was later applied by Zeleny (1982) and Hall (1989), and further developed by Yoon (1987), as well as Hwang, Lai, and Liu (1993).

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The method is particularly suitable for problems in a fuzzy environment that involve multiple criteria, a small number of decision-makers, and groups of alternatives.

The importance weights of different criteria and the significance levels of alternatives with respect to these criteria are considered as linguistic variables.

In order to evaluate the importance of each criterion and the performance of alternatives with respect to different criteria, decision-makers use linguistic variables. These linguistic variables can be expressed as positive triangular fuzzy numbers, as shown in Table 1 and Table 2 [Chen, 2000].

A triangular fuzzy number is represented as (l/m, m/u) or (l, m, u), which respectively denote the lowest possible value, the most probable (or mean) value, and the highest possible value in a fuzzy event where l is the lowest possible value, m is the most probable (modal) value, and u is the highest possible value

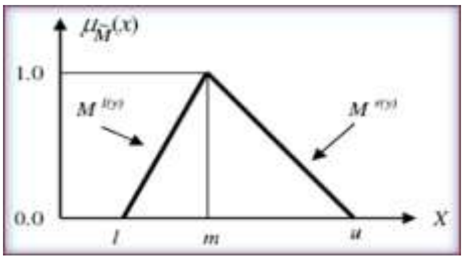

Figure 1. Triangular Fuzzy Number.

The linear representation of a triangular fuzzy number based on its left and right membership degree values is as follows:

$$\mu\left(x|\tilde{M}\right)=\begin{cases}0, & x<l\\ (x-l)/(m-l), & l\le x\le m,\\ (u-x)/(u-m), & m\le x\le u,\\ 0, & x>u\end{cases}$$

2.2 Steps of the Method

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Step 1: Determination of the decision-maker group, alternatives, and evaluation criteria

Initially, m alternatives (suppliers) are denoted as Ai=(1,2,3,…,m) and they are evaluated according to n selection criteria Cj=(1,2,3,…,n)

Subjective evaluations are performed by the decision-makers using linguistic terms. Based on these evaluations, the weight vector W=(w1,w2,…,wn) and the decision matrix X={xij,i=1,2,…,m;j=1,2,…,n} are established.

The weight vector WW represents the relative importance of the n selection criteria Cj=(1,2,3,…,n) for the given problem [Yurdakul and Iç, 2003].

Decision-Maker Group (Dk): D1, D2, D3 and k: number of decision-makers

Alternatives: A1, A2, A3

Evaluation Criteria (Cj):

C1 – Cost: Cost and payment terms are among the primary criteria that influence firms in selecting among alternatives. Suppliers attempt to become preferable through different pricing strategies and payment options.

C2 – Operational Convenience: The ease of using the shuttle system contributes to achieving the desired operational efficiency. The software used must also be easy to understand.

C3 – Flexibility: The system should allow for the storage of different types of teams with varying physical characteristics. It must accommodate technical specifications such as width, height, and load capacity in different storage configurations.

C4 – Reliability: The reliability of the shuttle depends on the correct operation of its software, its working capacity, and precision. These performance levels must be maintained consistently over many years.

C5 – Safety: The shuttle must be equipped with features that protect both itself and its surroundings from potential hazards during operation (e.g., stopping automatically when a worker extends their arm into its path).

C6 – After-Sales Service: Shuttle machines are high-cost and long-term investments. The return on investment depends on the duration during which the machine can be used without

hus, after-sales services such as training, spare part ty, and support offered by the supplier must be considered during selection.

Determination of the hierarchical structure of the

Figure 2. Hierarchical Structure of the Criteria.

Step 3: Construction of linguistic variables for weighting the criteria and linguistic scores for evaluating the alternatives

Table 1. Linguistic Variables and Their Corresponding Triangular Fuzzy Numbers

Linguistic Term	Triangular Fuzzy Number
Very Poor (VP)	(0, 0, 1)
Poor (P)	(0, 1, 3)
Medium Poor (MP)	(1, 3, 5)
Medium (M)	(3, 5, 7)
Medium Good (MG)	(5, 7, 9)
Good (G)	(7, 9, 10)
Very Good (VG)	(9, 10, 10)

Table 2. Linguistic Scores and Their Corresponding Triangular Fuzzy Numbers

Linguistic Term	Triangular Fuzzy Number (TFN)
Very Low (VL)	(0, 0, 1)
Low (L)	(0, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.9)
High (H)	(0.7, 0.9, 1)
Very High (VH)	(0.9, 1, 1)

Step 4: Evaluation of the criteria and alternatives by the decision-makers

The decision matrix X={xij, i=1,2,…,m; j=1,2,…,n} represents the performance ratings of each alternative Ai with respect to each selection criterion Cj.

Based on the provided weight vector and decision matrix, a ranking of all alternatives is obtained in line with the objective of the decision-making problem.

$$A_{ij} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

In the matrix Aij , m represents the number of decision alternatives, and n denotes the number of evaluation criteria (or factors).

For each main criterion, the weight values are calculated based on its associated sub-criteria. These calculated values are then used as the weights of the selection criteria. Since the values used are triangular fuzzy numbers, the resulting weights are also expressed as triangular fuzzy numbers.

Following the determination of the weight vector, the decision matrix is constructed. The decision matrix is derived from the evaluation of each criterion in terms of the alternatives. In this matrix, the assessments are made based on performance, and each criterion is evaluated for each alternative with respect to its relevant sub-criteria

Table 3. Evaluation of the Criteria.

	D1	D2	D3
C1	VH	VH	VH
C2	MH	VH	H
C3	H	MH	ML
C4	ML	MH	L
C5	ML	L	ML

Table 4. Decision-Maker's Evaluation Matrix of Alternative Methods Based on Criteria Using Linguistic Variables.

	C1			C2			C3			C4			C5			C6			
	D1	D2	D3	D1	D2	D3	D1	D2	D3	D1	D2	D3	D1	D2	D3	D1	D2	D3	
A1	VG	G	G	G	G	VG	G	VG	G	G	P	G	P	M	G	M	G	MG	
A2	G	MG	MG	VG	VG	G	MG	G	MG	M	G	MG	VG	MG	G	MG	M	MG	
A3	MG	VG	G	G	MG	VG	VG	MG	G	MG	MG	VG	MG	G	M	VG	MG	G	

Step 5: Conversion of the Decision-Makers' Alternative Evaluation Matrix into the Fuzzy Triangular Number System and Construction of the Normalized Fuzzy Decision Matrix

As a result of the alternative evaluations made in terms of fuzzy performance assessment, the total weights are obtained. These values are then subjected to a normalization process. Normalization is a mathematical operation performed to scale each criterion within the range [0,1], allowing for the comparison of results. The Standard Decision Matrix (R) is calculated by utilizing the elements of matrix A and using the following formula [Conurach and Cıobanu].

$$r_{ij} = \frac{a_{ij}}{\sqrt{\sum_{k=1}^{m} a_{kj}^2}}$$

The matrix R is obtained as follows:

$$R_{ij} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

First, the weight values for the evaluation factors (w_i) are determined:

$$\sum_{i=1}^{n} w_i = 1$$

Then, the elements in each column of the matrix R are multiplied by the corresponding weight values to form the matrix V. The matrix V is shown below:

$$V_{ij} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_n r_{1n} \\ w_1 r_{21} & w_2 r_{22} & \cdots & w_n r_{2n} \\ . & & & . \\ . & & & . \\ . & & & . \\ w_1 r_{m1} & w_2 r_{m2} & \cdots & w_n r_{mn} \end{bmatrix}$$

Table 5. Fuzzy Decision Matrix.

	C1			C2			C3		
Weight	0.9	1	1	0.7	0.87	0.97	0.43	0.63	0.80
A1	7.67	9.33	10	7.67	9.33	10	7.67	9.33	10
A2	5.67	7.67	9.33	8.33	9.67	10	5.67	7.67	9.33
A3	7	8.67	9.67	7	8.67	9.67	7	8.67	9.67
	C4			**C5**			**C6**		
Weight	0.2	0.37	0.4	0.07	0.23	0.43	0.7	0,87	0,97
A1	4.67	6.33	7.67	3.33	5	6.67	5	7	8.67
A2	3	7	8.67	7	8.67	9.67	4.33	6.33	8.33
A3	6.33	8	9.33	5	7	8.67	7	8.67	9.67

Table 6. Normalized Decision Matrix.

	C1			C2			C3		
A1	0.77	0.93	1	0.77	0.93	1	0.77	0.93	1
A2	0.57	0.77	0.93	0.83	0.97	1	0.57	0.77	0.93
A3	0.7	0.87	0.97	0.7	0.87	0.97	0.7	0.87	0.97
	C4			**C5**			**C6**		
A1	0.5	0.68	0.82	0.35	0.52	0.69	0.52	0.72	0.9
A2	0.32	0.75	0.93	0.72	0.9	1	0.45	0.65	0.86
A3	0.68	0.86	1	0.52	0.72	0.9	0.72	0.9	1

Table 7. Weighted Normalized Fuzzy Decision Matrix.

	C1			C2			C3		
A1	0.69	0.93	1	0.54	0.81	0.97	0.14	0.37	0.64
A2	0.52	0.77	0.93	0.58	0.84	0.97	0.25	0.49	0.74
A3	0.63	0.87	0.97	0.49	0.76	0.94	0.31	0.55	0.77
	C4			C5			C6		
A1	0.1	0.25	0.33	0.025	0.12	0.3	0.37	0.63	0.88
A2	0.064	0.28	0.38	0.05	0.21	0.43	0.32	0.56	0.83
A3	0.14	0.32	0.4	0.037	0.17	0.39	0.5	0.78	0.97

Step 6: Identification of the Fuzzy Positive (A^*) and Negative Ideal (A^-) Points, and Formation of the Solution Sets.

The TOPSIS method assumes that each evaluation factor has either a monotonically increasing or decreasing trend.

In order to construct the ideal solution set, the largest values (or the smallest, if the corresponding evaluation factor is minimization-oriented) of the weighted evaluation factors in the V matrix, i.e., the column values, are selected. The identification of the ideal solution set is shown in the following formula:

$$A^* = \left\{ (\max_i v_{ij} \middle| j \in J), (\min_i v_{ij} \middle| j \in J' \right\}$$

$$A^* = \left\{ v_1^*, v_2^*, \ldots, v_n^* \right\}$$

The negative ideal solution set is formed by selecting the smallest values (or the largest, if the corresponding evaluation factor is maximization-oriented) of the weighted evaluation factors in the V matrix, i.e., the column values. The identification of the negative ideal solution set is shown in the following formula:

$$A^- = \left\{ (\min_i v_{ij} \middle| j \in J), (\max_i v_{ij} \middle| j \in J' \right\}$$

$$A^- = \left\{ v_1^-, v_2^-, \ldots, v_n^- \right\}$$

In both formulas, denotes benefit (maximization), while denotes loss (minimization). Both the ideal and negative ideal solution sets consist of m elements, corresponding to the number of evaluation factors.

Step 7: Calculation of Separation (Distance) Measures

In the TOPSIS method, the Euclidean Distance Approach is used to calculate the deviations of each decision point's evaluation factor value from the Ideal and Negative Ideal solution sets. The deviation values obtained for the decision points are referred to as Ideal Separation and Negative Ideal Separation measures.

Ideal Separation (S_i^*) Measure:

$$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^*)^2}$$

Ideal Separation (S_i^-) Measure:

$$S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}$$

The number of S_i^* and S_i^- to be calculated here will naturally be equal to the number of decision points.

$S_1^* = 0.4065$ $S_1^- = 0.3251$
$S_2^* = 0.6124$ $S_2^- = 0.7764$
$S_3^* = 1.3195$ $S_3^- = 1.4466$

Step 8: Calculation of the Similarity/Closeness to the Positive Ideal Solution

The relative closeness of each decision point to the ideal solution is calculated using the ideal and negative ideal separation measures. The criterion used here is the proportion of the negative ideal separation measure within the total separation measure. The calculation of the relative closeness to the ideal solution is shown in the following formula:

$$C_i^* = \frac{S_i^-}{S_i^- + S_i^*}$$

C1* = 0. 4445
C2* = 0.5591
C3* = 0.5223

Step 9: Ranking of Alternatives Based on the Decreasing Closeness Coefficients to Determine the Preference Order.

Table 8. Formation of Solution Sets.

	S_i^*	S_i^-	C_i^*
A1	0,4065	0,3251	0,4445
A2	0,6124	0,7764	0,5591
A3	1,3195	1,4466	0,5223

As observed, A2 has the highest similarity/closeness coefficient and is therefore considered the most suitable shuttle model to be selected.

3. CONCLUSION

In this study, the Fuzzy TOPSIS method was successfully applied to address transportation-related problems in industrial environments, specifically in the selection of the most suitable shuttle system. By incorporating both qualitative and quantitative criteria such as cost, operational convenience, flexibility, reliability, safety, and after-sales service, the method allowed for a comprehensive evaluation of multiple alternatives under uncertainty.

The use of fuzzy logic enabled the transformation of subjective expert opinions into a structured decision-making framework. As a result of the analysis, Alternative A2 was found to have the highest closeness coefficient to the fuzzy positive ideal solution, indicating its superiority among the evaluated options.

This study demonstrates that the Fuzzy TOPSIS method is an effective and practical tool for solving complex multi-criteria decision-making (MCDM) problems in uncertain and dynamic industrial settings. The approach can be adapted to similar selection problems in logistics, manufacturing, and supply chain management, offering a robust foundation for informed and rational decision-making.

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