International Journal of Science and Engineering Applications
Volume 14-Issue 09, 33 – 38, 2025, ISSN:- 2319 - 7560
DOI: 10.7753/IJSEA1409.1005

Three-Dimensional Gravity-Magnetic Joint Inversion Based on Cross-Gradient Constraints

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[Abstract] The combined gravity-magnetic inversion is an effective means to improve the resolution of underground medium imaging and reduce the multi-solution problem of inversion. This paper systematically studies the three-dimensional gravity-magnetic combined inversion method based on the cross-gradient structure coupling. By introducing the cross-gradient operator to construct structural consistency constraints, the collaborative optimization of gravity and magnetic data in the inversion process is achieved. The theoretical model experiments show that this method can effectively enhance the focusing and structural consistency of the inversion results, especially in models with complex anomaly body shapes and significant differences in physical properties. Compared with the single property inversion method, the cross-gradient method significantly improves the spatial positioning ability of the anomaly body while maintaining the rationality of the property values.

[Key words] Cross-gradient; Gravity-magnetic combined inversion; Structural coupling; Three-dimensional inversion

1. INTRODUCTION

Geophysical inversion is an important technical means for inferring the underground physical property distribution based on surface observation data. However, a single geophysical method, due to its inherent multiple solutions and resolution limitations, is difficult to accurately depict complex geological structures. Joint inversion, by integrating multiple geophysical data and leveraging their complementarity, can significantly improve the reliability and resolution of the inversion results [1]. Gravity and magnetic methods exploration, due to their low cost, high efficiency, and convenient data acquisition, are often used in joint inversion research [2].

The joint inversion was first proposed by Vozoff et al. to solve the problems of direct current method and frequency-domain electrical method. If two methods sensitive to the same physical property can be combined, then the inversion resolution and reliability will be enhanced, resulting in better inversion results. Subsequently, joint inversion for different physical property parameters also began to emerge. Due to the low collection cost and high efficiency of gravity and magnetic geophysical data, geophysicists have conducted particularly rapid research on gravity-magnetic joint inversion [3]. Gravity-magnetic joint inversion methods can be divided into two main categories. One is to assume that the abstracted physical property models of the same geological structure have structural correlations, such as the joint inversion method proposed by Haber based on structural coupling; however, the operator calculation process of this method is very complex and its practicality is low [4]. Later, Gallardo et al. proposed the cross-gradient operator to construct the algorithm for structural coupling inversion and applied it to the joint inversion of electrical and seismic travel time [5]. Because the cross-gradient operator simplifies the calculation process of traditional structural coupling joint inversion and has good performance measurement and no singularities in the full space, cross-gradient joint inversion has been applied to the joint inversion of various geophysical data, such as Fregoso et al. using the cross-gradient method to conduct joint inversion of gravity and magnetic data [6]; Gallardo et al. conducting cross-gradient joint inversion of gravity, magnetic, and seismic travel time data [7]; Shi Xueming using cross-gradient joint inversion method to conduct joint inversion of cross-hole electromagnetic waves and seismic wave CT [8]; Ma Guoqing et al. conducting cross-gradient joint inversion of gravity and gravity gradient, etc. [9]. The other category is to construct mathematical models of the physical property relationships based on a large amount of statistical information of different physical properties to convert the underground medium, such as Sun et al. introducing prior information of rock properties into the regularization inversion by using the fuzzy C-means clustering method; Guo Man conducting Bayesian joint inversion of geoelectromagnetic and gravity data with rock property constraints; Liu Jie et al.

conducting further in-depth research and summarization on the coupling relationship between velocity and density in the gravity-seismic joint inversion [10]. In conclusion, gravity-magnetic joint inversion methods can fully utilize the complementarity of gravity-magnetic geophysical data, thereby improving the resolution and accuracy of the inversion, and thus has become the current research focus.

2. FORWARD MODELING AND INVERSE MODELING

2.1 Forward modeling of gravity anomalies

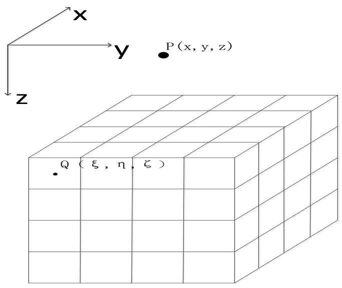

Figure 1: Schematic diagram of underground grid cell division

The forward calculation of gravity anomaly is shown in Figure 1. Firstly, the underground model is uniformly divided into multiple regular rectangular prism cells. Then, the anomaly of each rectangular prism cell for each observation point on the observation plane is calculated. Finally, by summing up the influence of all rectangular prism cells, the anomaly value of the entire underground model on the observation plane can be obtained.

The formula for calculating the gravity anomaly of the observation point P(x,y,z) corresponding to a single rectangular prism cell (Equation 1) is as follows:

$$\Delta g = -G_0\rho\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}(-1)^{(i+j+k)}\left[x_i In(y_i+\gamma_{ijk}) + y_i \ln(x_i+\gamma_{ijk}) + z_k \arctan(\tfrac{z_k\gamma_{ijk}}{x_i y_j})\right]$$

Where $x_i = x-\xi_i$, $y_i = y-\eta_j$, $z_k = z-\zeta_k$, $\gamma = (x_i^2 + y_j^2 + z_k^2)^{1/2}$, G_0 are the universal gravitational constant, ρ is the density of the rectangular prism model cell, Δg is the gravity anomaly, and Q(ξ,η,ζ) is the center of the volume element of the divided cell in the figure.

2.2 Independent Inversion of Gravitational Anomalies and Magnetic Anomalies

The inversion process can be understood as knowing the observation data and predicting the location of the underground anomaly body, that is $\mathbf{m}=\mathbf{G}^{-1}\mathbf{d}$, where d is the gravitational anomaly data, G is the kernel matrix, G^{-1} is the generalized inverse of G, and m is the underground model. For the independent inversion of gravitational anomalies and magnetic anomalies, the discretized objective function can be written as:

$$\Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \Phi_m(\mathbf{m}) + \Phi_l(\mathbf{m})$$

Where $\Phi_d(\mathbf{m})$ is the data fitting term, $\Phi_m(\mathbf{m})$ is the model constraint term, $\Phi_l(\mathbf{m})$ is the smooth constraint term. By further expanding the above formula, we can obtain:

$$\Phi_d(\mathbf{m}) = \|\mathbf{G}_z\mathbf{m}-\mathbf{d}_0\|^2_{\mathbf{C}_d^{-1}}$$

$$\Phi_m(\mathbf{m}) = \|\mathbf{W}_z(\mathbf{m}-\mathbf{m}_0)\|^2_{\mathbf{C}_m^{-1}}$$

$$\Phi_l(\mathbf{m}) = \|\mathbf{D}_x\mathbf{W}_z\mathbf{m}\|^2_{\mathbf{C}_{lx}^{-1}} + \|\mathbf{D}_y\mathbf{W}_z\mathbf{m}\|^2_{\mathbf{C}_{ly}^{-1}} + \|\mathbf{D}_z\mathbf{W}_z\mathbf{m}\|^2_{\mathbf{C}_{lz}^{-1}}$$

Where $\| \|^2$ is the L2 norm, and $\mathbf{C}_d$ and $\mathbf{C}_m$ are the covariance matrices of the data constraint term and the model constraint term respectively, $\mathbf{G}_z$ is the weighted kernel matrix with depth, $\mathbf{d}_0$ is the observed data; $\mathbf{W}_z$ is the overall depth-weighted matrix, $\mathbf{m}_0$ is the reference model; $\mathbf{D}_x$ 、 $\mathbf{D}_y$ 、 $\mathbf{D}_z$ are the difference operator matrices in the three axial directions, and $\mathbf{C}_{lx}$ 、 $\mathbf{C}_{ly}$ 、 $\mathbf{C}_{lz}$ are their corresponding covariance matrices. Here, smooth constraint and physical property constraint are used to alleviate the non-uniqueness and instability problems in the inversion process. Then, the objective function is solved using the Gauss-Newton method iteratively to obtain the final inversion result.

3. CROSS-GRADIENT JOINT INVERSION ALGORITHM

The cross-gradient operator was proposed by Gallardo and Meju. Due to its mathematical simplicity, absence of singularities, and the need for no normalization, it has been widely applied in joint inversions of various geophysical data such as gravity, magnetism, electromagnetics, and seismology. This paper focuses on the joint inversion method of gravity and magnetism based on cross-gradient constraints, and verifies its effectiveness through theoretical models and measured data. The cross-gradient joint inversion method used in this paper is consistent with the cross-gradient joint inversion code mentioned in Dr. Zhou Junjie's graduation thesis. Finally, there is:

$$p_{k+1} = p_k - N_k^{-1} n_k + N_k^{-1} B_k^T t_k$$

Among them:

$$N = P_k^T G^T C_d^{-1} G P_k + W_z^T D^T C_{lp}^{-1} D W_z + W_z^T C_m^{-1} W_z$$

$$n = P_k^T G^T C_d^{-1} \left(\hat{g}\left(p_k \right) - d_0 \right) + W_z^T D^T C_{lp}^{-1} D W p_k + W_z^T C_m^{-1} W_z (p_k - p_0)$$

The conditions that are met are as follows:

$$\left[B_{pk} N_k^{-1} B_{pk}^T + \frac{\max\left(B_{pk} N_k^{-1} B_{pk}^T \right)}{\beta} I \right] t_k = B_{pk} N_k^{-1} n_k - \tau_k$$

The partial derivative matrix is:

$$B_p = BP$$

4. TEST RESULTS

4.1 Example 1: Single Anomaly Body Model

In this subsection, the single anomaly body physical property model selected is located in the middle of the grid, with a burial depth of 100m, a north, east, and vertical extension of 200×200×150m, the remaining density of the anomaly body is 2800kg/m³, and the corresponding magnetic susceptibility is 1680e-5(SI). The inclination and deviation angles of the geomagnetic field are 50° and -5° respectively. The magnetization direction of the magnetic body is the same as the geomagnetic field (without the influence of remanent magnetism). The observation data area selected is 1950m*1950m, with 40*40 observation points, and the spacing between adjacent observation points is 50m. The underground model discussed in this section is located at the center of the observation data area. After forward modeling of this model, the observation data shown in Figure 2 (a) and (b) are obtained. Since this model has symmetry in the north-south direction, a vertical section at Northing = 500m can be selected, as shown in Figures 2 (c) and (d), and a cross-section at depth = 150m can be compared, as shown in Figures 2 (e) and (f).

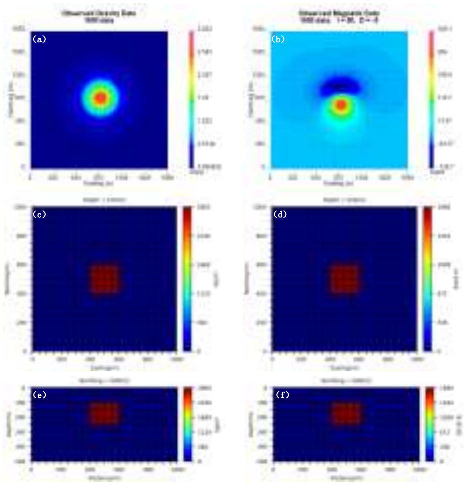

Figure 2 Single anomaly body model

(a) Gravity data; (b) Magnetic survey data; (c) Horizontal slice of the density model;

(d) Horizontal slice of the magnetic susceptibility model; (e) Vertical slice of the density model; (f) Vertical slice of the magnetic susceptibility model

Figure 3 shows the inversion results of gravity anomalies using either gravity anomaly alone or cross-gradient combined inversion. It can be seen from the figure that compared with the inversion using gravity anomaly alone, the inversion results obtained by the cross-gradient combined inversion method are more focused, the positions are more accurate, the model convergence is higher, and the physical property parameters are closer to the real values.

Figure 4 shows the inversion results of gravity anomalies using either magnetic anomaly alone or cross-gradient combined inversion. Similar results can be observed from Figure 4. The results in Figures 3 and 4 indicate that the method proposed in this paper is superior to the inversion using gravity/magnetic alone in overall effect.

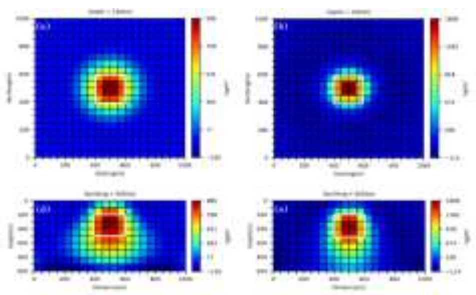

Figure 3 Gravity inversion results of the single anomaly body

model

(a), (d) Horizontal and vertical profiles of the density model obtained by gravity anomaly inversion alone;

(b), (e) Horizontal and vertical profiles of the density model obtained by the combined inversion of cross-gradient gravity and magnetic data;

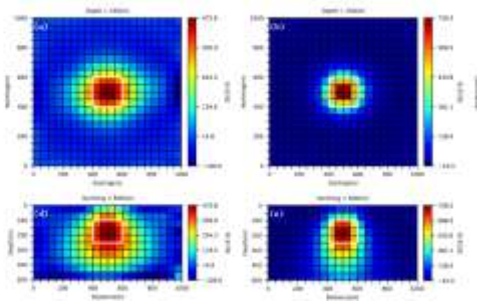

Figure 4 Magnetic inversion results of single anomaly body model

(a), (d) Horizontal and vertical cross-sections of the magnetization model obtained by single inversion of the magnetic anomaly;

(b), (e) Horizontal and vertical cross-sections of the magnetization model obtained by the combined inversion of cross-gradient gravity and magnetic data;

4.2 Case Study Two: Single property model of double anomaly bodies

The double anomaly single property model selected in this subsection is located in the middle of the grid. The distance between the two anomaly bodies is 200m, the burial depth is 100m, and the north, east, and vertical extensions are 200×150×150m. The residual density of the anomaly bodies is all 2800kg/m³, and the corresponding magnetization rates are all 1680e-5(SI). The inclination and deflection angles of the geomagnetic field are 50° and -5° respectively. The underground model we are discussing is located at the center of the observation data area, and the inversion results obtained after the forward modeling are shown in Figure 5 (a), (b). Since this model has symmetry along the north-south direction, the vertical section at Northing = 500m can be selected, as shown in Figure 5 (c), (d), and the cross-section at depth = 150m can be compared with it, as shown in Figure 5 (e), (f).

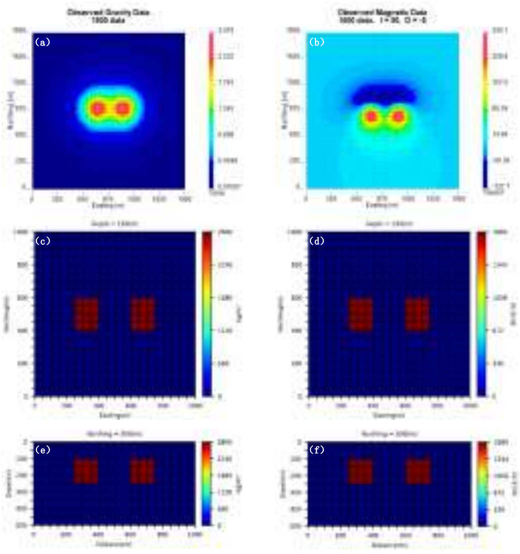

Figure 5 Dual anomaly body single density model

(a) Gravity data; (b) Magnetic survey data;

(c) Horizontal slice of the density model; (d) Horizontal slice of the magnetic susceptibility model; (e) Vertical slice of the density model; (f) Vertical slice of the magnetic susceptibility model

Figure 6 shows the inversion results of gravity anomalies alone and the joint inversion using cross-gradient. From the figure, it can be seen that under the same observation data, the joint inversion results using the method in this paper are more in line with the true physical properties in terms of material properties, and can separate two similar anomaly bodies horizontally. The convergence degree of the model is higher, and the inversion results are more focused and the position is more accurate.

Figure 7 shows the inversion results of magnetic anomalies alone and the joint inversion using cross-gradient. From Figure 7, similar results can be seen. The results in Figures 6 and 7 indicate that the overall effect of this method is better than the inversion using gravity/magnetic alone and the joint inversion using cross-gradient of gravity/magnetic.

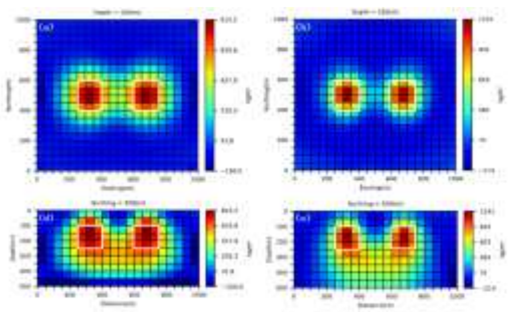

Figure 6 Results of single-property gravity inversion for double anomalies

(a), (d) Horizontal and vertical profiles of the density model obtained by single inversion of gravity anomalies;

(b), (e) Horizontal and vertical profiles of the density model obtained by cross-gradient gravity-magnetic joint inversion;

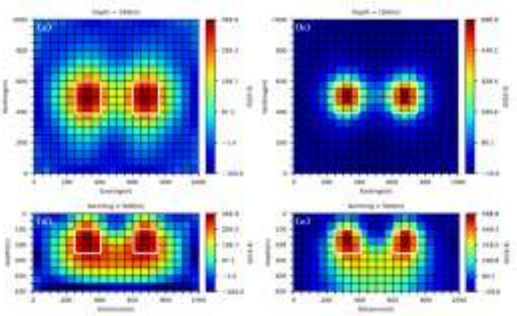

Figure 7 Results of single-property magnetic inversion with double anomalies

(a), (d) Horizontal and vertical profiles of the magnetization model obtained by single-property magnetic anomaly inversion;

(b), (e) Horizontal and vertical profiles of the magnetization model obtained by the cross-gradient gravity combined inversion;

5. CONCLUSION AND OUTLOOK

This paper systematically studies the three-dimensional gravity-magnetic combined inversion method based on the coupling of cross-gradient structure. Through theoretical model simulation and practical data application, the following main conclusions are drawn: The cross-gradient operator can effectively force the gravity-magnetic model to be consistent in the geometric structure, significantly improving the spatial coordination of the inversion results, especially in complex geological structure areas. Compared with single-property inversion, the cross-gradient combined inversion significantly reduces the multi-solution nature of the inversion process and improves the uniqueness and reliability of the results. In the double anomaly body model, this method can clearly distinguish adjacent anomaly bodies, accurately reflect their morphology, burial depth, and property differences, and has strong spatial resolution capability.

Although the cross-gradient combined inversion method proposed in this paper has achieved good results, there are still some issues and future development directions that deserve further research: In the future, the cross-gradient method can be extended to the joint inversion of more geophysical fields (such as electromagnetic, seismic, etc.) to achieve true multi-physical field collaborative inversion, further improving the comprehensiveness and accuracy of underground structure imaging. By combining intelligent optimization algorithms such as genetic algorithms and particle swarm optimization, the selection process of cross-gradient weight parameters can be optimized to improve inversion efficiency and global convergence. The introduction of deep learning technology can learn a large amount of prior geological information through neural networks, providing more reasonable initial models and constraints for cross-gradient inversion, further improving the inversion accuracy.

In conclusion, the cross-gradient-based gravity-magnetic combined inversion method has strong theoretical significance and practical value. Through continuous improvement and expansion, it is expected to become an important tool in underground resource exploration and geological structure research. Future research will continue to deeply explore its applicability in different geological environments, different data types, and different application scenarios, promoting the method to a more mature and widely applied stage.

6. REFERENCES

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