

Performance Analysis of MUSIC Algorithm and Its Modified Algorithm

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Abstract: The Multiple Signal Classification (MUSIC) algorithm, as a core branch algorithm in the field of array signal processing, has become a key research direction in array direction finding and signal parameter estimation. This study systematically investigates the impact of core parameters such as element spacing, snapshots, and signal-to-noise ratio on the performance of the MUSIC algorithm by adjusting these parameters and combining spectral peak feature analysis with root mean square error quantification. To address the issue of algorithm failure in complex environments, the study proposes three improvement methods, effectively enhancing the algorithm's performance.

Keywords: MUSIC; Array signal processing; spectral peak; root mean square error; Algorithm performance;

1. INTRODUCTION

This study examines the performance of the MUSIC algorithm by varying the element spacing, number of snapshots, and signal-to-noise ratio. It also compares Root MUSIC, weighted MUSIC, and the improved MUSIC algorithm with conventional MUSIC using spatial-spectrum peaks and root mean square error (RMSE).

2. RESEARCH BACKGROUND

When the number of array elements is limited and only one signal source is present, the source azimuth can be obtained using conventional mathematical methods. However, when multiple sources are present, these methods are no longer sufficient, and the MUSIC algorithm is introduced for direction-of-arrival (DOA) estimation. Its performance may deteriorate in challenging conditions, including low SNR, spectral leakage, coherent sources, and an unknown number of sources. To address these limitations, Root MUSIC, weighted MUSIC, and an improved MUSIC algorithm have been developed, each targeting different application scenarios.

3. METHODOLOGY

3.1 MUSIC Algorithm

Consider a uniform linear array (ULA) consisting of M elements with an inter-element spacing of d . Each element receives the transmitted signals from D independent sources.

Under these conditions, the array signal model is given by:

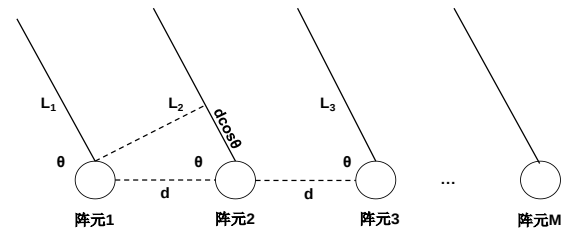


Figure 1. Array signal model

The total signal received by the m -th array element, denoted by $x_m(t)$, is the superposition of the source signals and additive noise:

$$x_m = \sum_{k=1}^D s_k(t) e^{-j2\pi \frac{(m-1)d \sin \theta_k}{\lambda}} + n_m(t) \quad (1)$$

where:

$S_k(t)$ is the complex envelope of the k -th source;

$n_m(t)$ is additive white Gaussian noise independent of the source signals;

In vector-matrix form:

$$S = [s_1(t) \quad s_2(t) \quad \cdots \quad s_M(t)]^T \quad (2)$$

$$A = [\alpha(\theta_1) \quad \alpha(\theta_2) \quad \cdots \quad \alpha(\theta_D)] \quad (3)$$

$$a(\theta_k) = \begin{bmatrix} 1 & e^{-j2\pi \frac{d \sin(\theta_k)}{\lambda}} & \cdots & e^{-j2\pi \frac{(M-1)d \sin(\theta_k)}{\lambda}} \end{bmatrix}^T \quad (4)$$

The MUSIC algorithm forms the covariance matrix of the received data and decomposes it into signal and noise subspaces. The spatial spectrum is then constructed from the noise subspace, and the DOAs are estimated by searching for its peaks.

Assuming that the target signals are independent of zero-mean Gaussian white noise with variance σ^2 , we have:

$$R_{XX} = E(XX^H) = AR_{SS}A^H + \sigma^2 I \quad (5)$$

Eigenvalue decomposition of the covariance matrix yields M eigenvalues and their corresponding eigenvectors:

(1) Signal subspace: The D largest eigenvalues correspond to the signal components, and the space spanned by their eigenvectors is identical to the space spanned by the steering matrix $A(\theta)$.

(2) Noise subspace: The remaining M-D eigenvalues correspond to the noise components and are theoretically equal to σ^2 . The associated eigenvectors are combined to form the noise matrix G_n :

$$G_n = [v_{D+1} \quad v_{D+2} \quad \cdots \quad v_M] \quad (6)$$

Matrix theory shows that the noise subspace is orthogonal to the steering vectors of the signal sources. Based on this property, the MUSIC spatial spectrum is defined as:

$$P_{MUSIC}(\theta) = \frac{1}{a^H(\theta)G_n G_n^H a(\theta)} \quad (7)$$

In Equation (9), the denominator represents the inner product between the steering vector and the noise matrix. At the true DOA, the denominator approaches zero in the ideal case. In practice, noise makes it nonzero but small, so the reciprocal produces a spectral peak. The DOAs can therefore be estimated by locating the spectral peaks.

3.2 Root MUSIC Algorithm

Root MUSIC eliminates the need for spectral peak searching by replacing it with polynomial rooting. Define the polynomial as:

$$f(z) = z^{K-1} p^T(z^{-1}) U_N U_N^H p(z) \quad (8)$$

The azimuth estimate is then given by:

$$\theta = \arccos\left[\frac{1}{2\pi d} \arg(z_i)\right] (i=1,2,\dots,N) \quad (9)$$

Compared with conventional MUSIC, Root MUSIC is particularly suitable in the following situations:

(1) At high SNR, Root MUSIC provides performance comparable to conventional MUSIC while reducing computational complexity because it does not require spectral peak searching.

(2) Root MUSIC can offer better robustness than conventional MUSIC in colored-noise environments.

3.3 Weighted MUSIC Algorithm

In array-signal processing, the equal eigenvalues associated with the noise subspace can be used to retain or emphasize selected noise eigenvectors [2]. Conversely, the relatively large eigenvalues associated with the signal subspace can be down-weighted to reduce the contribution of the signal eigenvectors. The resulting eigenvector space approximates the noise subspace, while preserving subspace orthogonality and avoiding eigenvalue sorting.

The spatial spectrum of weighted MUSIC is expressed as:

$$P_{WMUSIC} = \frac{1}{a^H(\theta) U U^H a(\theta)} \quad (10)$$

Weighted MUSIC is applicable in the following situations:

(1) When high source-localization accuracy is required, such as in radar and wireless communications, an eigenvalue-weighting matrix can improve the estimation accuracy. The weighting matrix can also mitigate multipath interference and spectral leakage.

(2) When the number of sources is unknown, weighted MUSIC can perform estimation without prior knowledge of the source number.

3.4 Improved MUSIC Algorithm

For coherent sources, the covariance matrix becomes rank-deficient, making the signal and noise subspaces difficult to separate. This study adopts a dimension-reduction method based on the j-transform.

The j-transform converts real signals into complex representations and improves the covariance matrix used for subspace decomposition. The improved MUSIC algorithm therefore retains the array aperture and improves DOA estimation for coherent sources.

4. RESULTS

4.1 MUSIC Algorithm

4.1.1 Effect of Element Spacing

Simulation conditions: eight array elements, five signal sources, source directions of -60° , -30° , 0° , 30° , and 60° , an SNR of 10 dB, and 500 snapshots.

When the element spacing does not exceed 0.5 wavelengths, the number of dominant eigenvalues approaches the number of sources as the spacing increases. At an element spacing of 0.5 wavelengths, the number of dominant eigenvalues exactly matches the number of sources, as shown in the figure.

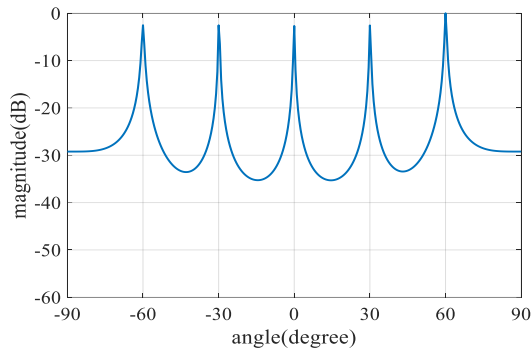


Figure 2. Element spacing of 0.5λ and RMSE of 7.94

Five distinct peaks can be observed, and the RMSE at 60° is 7.94. The algorithm therefore achieves its best performance under these conditions [1].

When the element spacing exceeds 0.5 wavelengths, the number of dominant eigenvalues is less than five, whereas more than five peaks appear in the spectrum. These additional peaks are referred to as false peaks. An excessively large spacing introduces spatial aliasing because multiple phase values may satisfy the steering-vector orthogonality condition, producing grating lobes. The spacing should therefore generally be no greater than 0.5 wavelengths.

When the element spacing is less than 0.5 wavelengths, fewer than five dominant eigenvalues and spectral peaks are observed, a phenomenon referred to as peak loss. With overly small spacing, the phase differences between the signals received by different elements become too small to resolve the sources accurately. Spatial sampling is also insufficient, which further reduces the ability to capture the signal characteristics.

4.1.2 Effect of Signal-to-Noise Ratio

Simulation conditions: eight array elements, five signal sources at -60° , -30° , 0° , 30° , and 60° , 500 snapshots, and an element spacing of 0.5 wavelengths.

As the SNR increases, the spectral peaks become sharper and the RMSE decreases, indicating improved algorithm performance. The higher SNR makes the signal characteristics more distinguishable from the noise, allowing MUSIC to produce more pronounced spectral peaks.

4.1.3 Effect of the Number of Snapshots

As the number of snapshots increases, the spectral peaks become sharper and the RMSE decreases. More observations provide a more accurate estimate of the signal parameters and covariance matrix, thereby improving DOA-estimation accuracy.

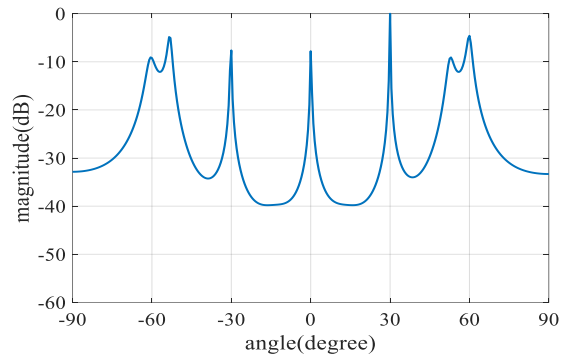


Figure 3. The element spacing is 0.6λ , and the root mean square error is 5.32

4.2 Comparison of Root MUSIC, Weighted MUSIC, and Conventional MUSIC

Simulation conditions: one signal source at -60° , an SNR of 10 dB, an element spacing of 0.5 wavelengths, 200 snapshots, and a varying number of array elements.

As the number of array elements increases, the RMSE of all three algorithms decreases, indicating improved performance. With a larger array, conventional MUSIC and Root MUSIC achieve lower RMSE than weighted MUSIC under the tested conditions. Root MUSIC and conventional MUSIC have similar performance, but Root MUSIC avoids spectral peak searching and therefore has lower computational complexity. Weighted MUSIC requires an appropriate weighting matrix, which increases both the design burden and computational complexity.

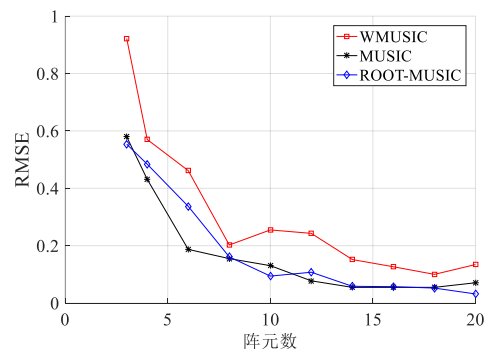


Figure 4. RMSE comparison of the three algorithms for different numbers of array elements

4.3 Improved MUSIC Algorithm

Simulation conditions: two coherent sources at 0° and 60° , an SNR of 10 dB, an element spacing of 0.5 wavelengths, 500 snapshots, and a signal frequency of $\pi/4$. All other conditions remain unchanged.

For coherent sources, the spectral peaks of conventional MUSIC at the preset directions are less pronounced than those of the improved MUSIC algorithm. Conventional MUSIC also

produces false peaks, or grating lobes, which interfere with DOA identification. It therefore fails under coherent-source conditions, whereas the improved algorithm provides better performance. This degradation occurs because coherent sources have similar spatial responses, which reduces the rank of the covariance matrix and weakens the separation between the signal and noise subspaces. The J-transform restores the effective rank of the covariance matrix to the number of sources and mitigates the effect of source correlation, enabling the improved MUSIC algorithm to estimate the DOAs of coherent sources.

5. CONCLUSIONS

The effects of element spacing, SNR, and the number of snapshots were examined through simulations. The main conclusions are as follows:

(1) An element spacing of approximately half the incident wavelength produces the sharpest spectral peaks and the lowest RMSE for conventional MUSIC. Spacing greater than 0.5 wavelengths causes grating lobes. Increasing the SNR and the number of snapshots generally improves performance, although the RMSE becomes stable after the number of snapshots exceeds a certain level.

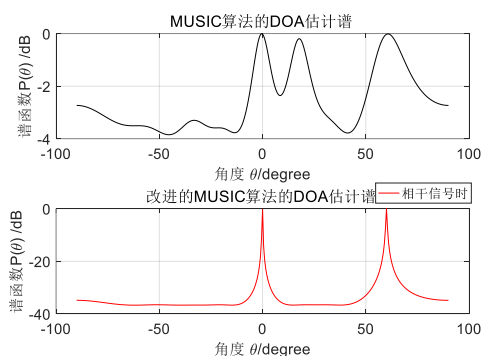


Figure 5. Spectral peaks of the two algorithms for coherent sources

(2) Under low-SNR conditions, weighted MUSIC and Root MUSIC achieve lower RMSEs than conventional MUSIC, indicating stronger noise robustness. At high SNR, the three algorithms produce similarly low RMSEs and comparable performance.

(3) Conventional MUSIC produces false and indistinct peaks for coherent sources and therefore fails in this scenario. The improved MUSIC algorithm produces sharper peaks and can effectively estimate the DOAs of coherent sources.

6. REFERENCES

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