International Journal of Science and Engineering Applications
Volume 15-Issue 12, 19 – 27, 2026, ISSN:- 2319 - 7560
DOI: 10.7753/IJSEA1512.1006

Robust Underdetermined DOA Estimation Method for Coprime Arrays under Non-uniform Noise

Liang Xie
Yangtze University, Jingzhou, China

Abstract: Aiming at the performance degradation of underdetermined DOA estimation methods based on coprime arrays under non-uniform noise conditions, this paper proposes a robust DOA estimation method based on covariance matrix reconstruction and matrix completion. Firstly, an iteratively weighted spatial-temporal covariance matrix estimation method is adopted to obtain an accurate covariance matrix. Then, the received data covariance matrix is decomposed to acquire a diagonal matrix containing non-uniform noise terms. Next, the minimum value among the diagonal elements is selected to replace the other diagonal elements, thereby obtaining the reconstructed data covariance matrix. Finally, the reconstructed covariance matrix is extended and processed by matrix completion, and the subspace method is combined to implement DOA estimation. Theoretical analysis and simulation results demonstrate that, compared with existing methods, the proposed method effectively suppresses the influence of non-uniform noise and achieves superior DOA estimation performance.
Keywords: Direction of Arrival Estimation; Coprime Array; Non-uniform Noise; Generalized Inner Product (GIP)

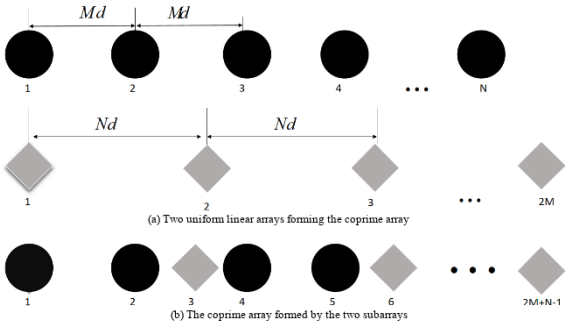
0 Introduction

Direction-of-arrival (DOA) estimation is widely used in radar signal processing, target detection, navigation and many other fields, and is one of the core research topics in array signal processing. In modern electromagnetic environments, the sources are dense, complex and time-varying, so that direction finding of multiple sources has to be achieved with a limited number of sensors. Most classical super-resolution algorithms are designed for uniform linear arrays, whose degrees of freedom are limited by the number of sensors: a uniform array with elements can provide at most degrees of freedom, and therefore underdetermined DOA estimation is difficult. To solve this problem, the coprime array was proposed in [1]. Compared with a conventional uniform array, a coprime array with sensors can achieve a much higher number of degrees of freedom, because its difference co-array contains a virtual uniform linear array of larger size. Since this framework was proposed, it has attracted extensive attention [2-9]. At present, the classical underdetermined DOA estimation algorithms for coprime arrays mainly include the spatial smoothing algorithm [10], sparse algorithms [11] and array interpolation algorithms [12]. All of these algorithms assume spatially white Gaussian noise, and their performance degrades severely when this assumption is violated. In practice, due to mutual coupling between sensors, non-ideal receive channels and array miscalibration, correlated colored noise frequently occurs. In general the structure of colored noise is unknown; however, in some cases, such as sparsely distributed sensors, colored noise can be further simplified to non-uniform noise [13-15]. N $N-1$ [1] $O\left(N^2\right)$

The main idea of DOA estimation under non-uniform noise is to estimate and whiten the noise covariance matrix under certain restrictions (for example, part of the noise information must be known, or the number of sensors must be at least three times the number of sources), after which a subspace-based algorithm is applied. Reference [16] proposed a spatial-smoothing based DOA estimation method, in which the noise covariance matrix is first obtained iteratively to eliminate the influence of noise on the array covariance matrix, then the rank of the covariance matrix is restored by spatial smoothing, and finally a subspace method is used. Reference [17] eliminated the influence of noise on the received data by an averaging operation and then proposed a weighted L1-norm sparse reconstruction method, whose advantage is that neither the noise covariance matrix nor the number of incident signals has to be known in advance. However, these methods usually sacrifice a large part of the array aperture and require the signal environment to be overdetermined, i.e., the number of sources must be smaller than the number of sensors. With the increasing complexity of modern electromagnetic environments, where radiation sources are numerous and densely distributed, this condition is not necessarily satisfied, and underdetermined DOA estimation (more sources than sensors) becomes increasingly common. Therefore, research on underdetermined DOA estimation under non-uniform noise is of great practical significance. Reference [18] pre-whitened the non-uniform noise to

International Journal of Science and Engineering Applications
Volume 15-Issue 12, 19 – 27, 2026, ISSN:- 2319 - 7560
DOI: 10.7753/IJSEA1512.1006

achieve robust DOA estimation, and reference [19] also pre-whitened the non-uniform noise, but these methods require the number of sensors to be at least



three times the number of sources. The methods in [17-19] all require more sensors than sources and are therefore not applicable to underdetermined DOA estimation. To this end, reference [20] proposed a compressed-sensing based DOA estimation algorithm that is applicable to coprime arrays, and reference [21] further extended it to wideband signals; however, this method directly removes the diagonal elements of the data covariance matrix, which degrades the performance. Reference [22] proposed a DOA estimation method for coprime arrays under non-uniform noise, but it only exploits the continuous virtual sensors of the difference co-array and discards the discontinuous part, so that the optimum performance cannot be achieved. Reference [23] addressed underdetermined DOA estimation in non-uniform noise and proposed an estimation method based on total-variation-norm minimization, which also fails to exploit the discontinuous virtual sensors. Reference [24] fully exploited all virtual sensors and suppressed non-uniform noise by replacing the diagonal elements with the minimum diagonal element; however, when the sample covariance matrix cannot be guaranteed to be accurate, directly replacing the main-diagonal elements also removes useful signal components from the diagonal, resulting in a loss of information. Therefore, this paper proposes a method that combines iteratively weighted covariance matrix estimation with matrix reconstruction and completion, which can effectively suppress non-uniform noise while preserving useful information.

1 Coprime Array Model

The coprime array model is shown in Fig. 1. The array consists of two linear uniform subarrays. Subarray 1 contains sensors with a spacing of , and subarray 2 contains sensors with a spacing of ; the two subarrays share the first sensor. Here and are coprime integers with , and is the wavelength of the incident signals. The set of sensor positions can then be expressed as $N\ M\lambda/2\ M\ N\lambda/2\ M\ N\ M<N$ $d_1=\lambda/2\ \ \lambda$

$$P=\{mN\lambda/2|\ m=0,1,\ldots,M-1\}\cup$$
$$\{nM\lambda/2|\ n=0,1,\ldots,N-1\}$$

Assume that far-field narrowband signals impinge on the coprime array from directions , respectively. The array output data can be expressed as: $\theta=[\theta_1,\theta_2,\ldots,\theta_K]$

$$X(t)=AS(t)+N(t) \qquad (1)$$
$$X(t)=\left[x_1(t),x_2(t),\ldots,x_{2M+N-1}(t)\right]^T$$

where is the array output data, is the incident signal vector, is the noise vector, is the manifold

Fig. 1 Schematic diagram of the coprime array1

matrix, is the steering vector, denotes the physical sensor positions of the coprime array, and is the position of the first sensor.

$$S(t)=\left[s_1(t),s_2(t),\cdots,s_L(t)\right]$$
$$N(t)=\left[n_1(t),n_2(t),\ldots,n_{2M+N-1}(t)\right]^T$$
$$A=\left[a(\theta_1),a(\theta_2)\cdots,a(\theta_L)\right]$$
$$a(\theta_l)=\left[1,e^{-j\frac{2\pi p_2}{\lambda}\sin\theta_l},\cdots,e^{-j\frac{2\pi p_{2M+N-N}}{\lambda}\sin\theta}\right]^T$$
$$p_i\in \mathrm{P}, i=1,2,\cdots,2M+N-1\ \ d_1=0$$

Assuming that the incident signals are mutually independent and uncorrelated, taking the expectation of the array output data gives the covariance matrix of the received data:

$$R_{xx}=\mathrm{E}\left\{X(t)X^{\mathrm{H}}(t)\right\}=AR_SA^{\mathrm{H}}+R_n \qquad (2)$$

where denotes the expectation operator, denotes the conjugate transpose, is the signal covariance matrix and is the noise covariance matrix. Let the noise power vector be . Under ideal conditions with a spatially white background, ; when the background noise is non-uniform, the noise powers are not equal and the noise covariance matrix is expressed as $E[\square]$

$$(\square)^{\mathrm{H}}\qquad R_S=\mathrm{E}\left\{S(t)S^{\mathrm{H}}(t)\right\}\qquad R_n=E\left\{N(t)N^{\mathrm{H}}(t)\right\}$$
$$\partial^2=\mathrm{E}\left\{\partial_1{}^2,\partial_2{}^2,\cdots,\partial^2{}_{2M+N-1}\right\}\ R_n=\partial^2 I$$
$$R_n=\mathrm{diag}\{\partial\}=\mathrm{diag}\left\{\partial_1^2,\partial_2^2,\cdots,\partial^2_{2M+N-1}\right\} \qquad (3)$$

where denotes a diagonal matrix. In practice the number of snapshots V is finite, and the true covariance matrix is replaced by the sample covariance matrix, i.e., $\mathrm{diag}\{\square\}$

International Journal of Science and Engineering Applications
Volume 15-Issue 12, 19 – 27, 2026, ISSN:- 2319 - 7560
DOI: 10.7753/IJSEA1512.1006

$$R_e = \frac{1}{V}\sum_{v=1}^{V} X(t_v)X^{\mathrm{H}}(t_v) \quad (4)$$

2 Robust DOA Estimation Method for Coprime Arrays under Non-uniform Noise

2.1 Covariance Estimation via Generalized Inner Product Iterative Weighting

Simulations show that the algorithm cannot guarantee estimation accuracy in the presence of sensor failures, a small number of sensors, or severely contaminated signals. Accurate and robust DOA estimation is therefore required, and the covariance matrix is estimated in the non-uniform noise environment as follows.

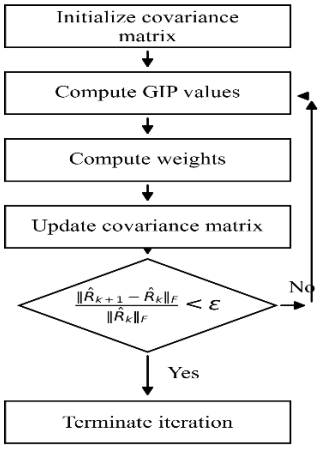


Fig. 2 Flowchart of the algorithm

The covariance matrix is estimated by the following weighting scheme:

(1) Initialize the covariance matrix: the conventional sample covariance matrix (SCM) method is used to estimate the initial clutter covariance matrix:

$$\hat{R}_0 = \frac{1}{M}\sum_{i=1}^{M} x_i x_i^{\mathrm{H}} \quad (5)$$

where M is the total number of training samples.

(2) Compute the generalized inner product (GIP) values and their probability distribution: first, the GIP value of each sample cell is computed from the obtained covariance matrix: z_i

$$z_i = x_i^{\mathrm{H}}\hat{R}_j^{-1}x_i, i = 1,2,\cdots,M \quad (6)$$

where is the clutter covariance matrix obtained at the k-th iteration, $\hat{R}_j$ j

The matrix is used in the initial iteration. The probability distribution of the GIP values is then obtained by histogram statistics. $\hat{R}_0$

$$P(z_i), i = 1,2,\cdots,M \quad (7)$$

(3) Compute the weights: the weight of each sample is obtained from the difference between its GIP value and the mean GIP value; therefore the key to a reasonable weighting is to obtain a mean GIP value that reflects the homogeneous data.

In general, homogeneous data can be assumed to constitute the majority of the training samples while non-homogeneous data are relatively few. Moreover, since the GIP values of homogeneous data are similar to each other and clearly different from those of non-homogeneous data, the probability of homogeneous data in the statistical distribution of GIP values should be larger than that of non-homogeneous data.

Based on the above analysis, in order to obtain a reasonable mean GIP value and avoid the influence of non-homogeneous samples, only the samples with high probability are used in the mean calculation.

$$z_{av} = \frac{1}{m}\sum_{i\in\theta} z_i \quad (8)$$

where is the number of elements in the set, $\theta = \{i | P(z_i) \geq p\}, m$ θ

$p = \mu_{P(z_i)}$ is the preset probability value, and is the mean value of . $\mu_{P(z_i)}$ $P(z_i)$

The weight of each sample is

$$c_i = \frac{1}{\sigma}\exp\left[-\frac{|z_i - z_{av}|}{z_{av}}\right], i = 1,2,\cdots,M \quad (9)$$

where is a normalization factor. σ $\sum_{i=1}^{M}\exp\left[-\frac{|z_i - z_{av}|}{z_{av}}\right]$

(4) Update the clutter covariance matrix: the estimated covariance is obtained by weighting the samples with the obtained weights

$$\hat{R}_j = \sum_{i=1}^{M} c_i x_i x_i^{\mathrm{H}} \quad (10)$$

When the estimated covariance matrix satisfies the condition $\frac{\|\hat{R}_{j+1} - \hat{R}_j\|_F}{\|\hat{R}_j\|_F} < \varepsilon$

the iteration terminates and the covariance matrix is obtained; otherwise the covariance matrix is substituted into step (2) for further iteration. $\hat{R}$

In summary, the GIP-probability-histogram based adaptive weighted iterative method adopted in this paper improves the accuracy of covariance matrix estimation and enhances the detection performance of the system.

International Journal of Science and Engineering Applications
Volume 15-Issue 12, 19 – 27, 2026, ISSN:- 2319 - 7560
DOI: 10.7753/IJSEA1512.1006

2.2 Data Preprocessing of the Difference Co-array

To analyze the virtual sensor characteristics of the coprime array intuitively, let the n-th sensor of subarray 1 be and the m-th sensor of subarray 2 be . Based on the set of physical sensor positions, the position set of the difference co-array is defined as: p_n q_m

$$D = \left\{ P_i - P_j \mid P_i, P_j \in P \right\} \quad (11)$$

where P is the set of physical sensor positions of the coprime array and D is the set of position differences, which contains repeated elements. A new set D̃ consisting of all distinct elements of D is formed as:

$$\tilde{D} = \left\{ d \mid d \in D \right\} \quad (12)$$

Let c(d) denote the number of times the element d appears in D. From the properties of coprime arrays, the elements of D̃ are the virtual sensor positions of the difference co-array, and the number of non-negative elements is the effective number of degrees of freedom of the coprime array, which directly determines the maximum number of sources that can be estimated.

Expanding Eq. (2), the element expression of the received data covariance matrix of the coprime array is obtained as:

$$R(i,j) = \mathrm{E}\left\{ X(t) X^{\mathrm{H}}(t) \right\} = A R_S A^{\mathrm{H}} + \partial^2 \delta_{i,j} \quad (13)$$

where is the Kronecker delta function, equal to when i=j and otherwise; denotes the element in the i-th row and j-th column of the covariance matrix. Further expanding the signal component gives: $\delta_{i,j}$

$\delta_{i,j} = 1$ $\delta_{i,j} = 0$ $R(i,j)$

$$[R](i,j) = \sum_{k=1}^{K} \partial_{s,k}^2 e^{-j2\pi(p_i - p_j)\sin\theta_k/\lambda} + \partial_s^2 \delta_{i,j} \quad (14)$$

where is the power of the k-th incident signal and is the phase term corresponding to the propagation path difference, i.e., the signal component of the covariance matrix. $\partial_{s,k}^2$

$e^{-j2\pi(p_i - p_j)\sin\theta_k/\lambda}$

From Eq. (11), the physical sensor position differences correspond to the virtual sensor positions of the difference co-array, and the set D̃ is obtained after removing repeated differences. Combining Eqs. (12) and (14), there is a one-to-one correspondence between the virtual sensor positions of the difference co-array and the elements of the received data covariance matrix. Due to the influence of non-uniform background noise, the covariance elements corresponding to the same virtual sensor position take different values, so they are averaged to reduce the noise interference: $p_i - p_j$

$$R_d(d) = \frac{1}{c(d)} \sum_{pi-pj} [R]_{i,j} \quad (15)$$

where is the averaged covariance element corresponding to virtual sensor position d, and c(d) is the number of covariance elements corresponding to position d. $R_d(d)$

2.3 Non-uniform Noise Suppression Based on Covariance Matrix Reconstruction

Non-uniform noise is mainly manifested as the distortion of the diagonal elements of the covariance matrix. To reduce its influence, the received data covariance matrix is first decomposed into the sum of the signal covariance matrix and the noise covariance matrix:

$$R = R_{sig} + R_n \quad (16)$$

where is the pure signal covariance matrix, whose off-diagonal elements are the cross-correlation terms of the signals and whose diagonal elements are the signal powers at the individual sensors; and is the non-uniform noise covariance matrix, which only affects the diagonal elements of R and whose diagonal elements are all real.

$R_s ig = A R_s A^H$ $R_n = diag\left(\sigma_1^2, \sigma_2^2, \ldots, \sigma_l^2\right)$

From Eq. (16), the diagonal elements of the covariance matrix are the sum of the signal power and the non-uniform noise power at the corresponding sensor, i.e., where is the signal power at the i-th sensor. To suppress the influence of non-uniform noise, the minimum diagonal element of the covariance matrix is chosen and denoted as

$[R](i,i) = \sigma(sig,i)^2 + \sigma_i^2$ $\sigma_{(sig,i)}^2$

$$R_{\min} = min\left\{ [R](1,1), [R](2,2), \ldots, [R](L,L) \right\}$$

This value is the superposition of the signal power at the corresponding sensor and the minimum noise power, and therefore has the lowest noise interference.

By replacing all diagonal elements of the covariance matrix with this value, the reconstruction is completed and the reconstructed covariance matrix is: $R_{\min}$

$$\hat{R} = R - diag\left(R_{1,1} - R_{\min}, R_{2,2} - R_{\min}, \cdots, R_{L,l} - R_{\min} \right) \quad (18)$$

The essence of covariance matrix reconstruction is to pre-whiten the non-uniform noise: by unifying the noise power at each sensor to the minimum noise power, the distortion caused by non-uniform noise is

effectively suppressed; at the same time, replacing the larger noise powers with the minimum noise power is equivalent to increasing the SNR at each sensor, which lays the foundation for the subsequent DOA estimation.

2.4 Full Exploitation of the Difference Co-array Based on Matrix Completion

Substituting the accurately estimated and reconstructed covariance matrix into Eq. (15) yields the covariance elements corresponding to the virtual sensors of the difference co-array. The difference co-array of a coprime array contains holes, i.e., some virtual sensor positions have no corresponding physical position difference. Conventional methods only use the uniform subarray formed by the continuous virtual sensors and discard the discontinuous part, so that the degree-of-freedom advantage of the coprime array is not fully exploited. $\hat{R}$ $\hat{R}_d(d)$

To fully exploit the information of all virtual sensors, the matrix is first extended by constructing a Toeplitz matrix of dimension , where is the total number of virtual sensors of the difference co-array and G is a selection matrix defined according to the coprime array sensor positions. The elements of the Toeplitz matrix satisfy row-shift invariance and are defined as: G $\hat{R}$ $N_V \times N_V$ R_E N_V

$$\left[T_{i,j}\right] = \hat{R}_d\left(d_{i-j}\right) \tag{19}$$

where is the position of the k-th virtual sensor of the difference co-array. In the extended Toeplitz matrix, the elements corresponding to the holes of the difference co-array are zero, i.e., the zero elements correspond one-to-one to the virtual sensor holes. d_{i-j} $i-j$

Matrix completion is performed on the zero elements of the Toeplitz matrix to recover the information of the difference co-array holes. The matrix completion problem can be described by the rank minimization model:

$$\min_{T^*} rank\left(T^*\right) \quad \text{s.t} \quad \mathrm{p}_\Omega\left(T^*\right) = \mathrm{p}_\Omega\left(T\right) \tag{20}$$

where is the target matrix to be completed, is the rank of the matrix, is the projection operator, and Ω is the set of positions of the non-zero elements, which is determined once the difference co-array is fixed. The projection operator is expressed as: T^* $rank(\cdot)$ $\mathrm{p}_\Omega(\cdot)$

$$\left[\mathrm{p}_\Omega\left(T\right)\right] = \begin{cases} \left[T\right]_{i,j}, & (i,j) \in \Omega \\ 0, & (i,j) \in \Omega \end{cases} \tag{21}$$

Since directly solving the rank minimization problem is NP-hard, nuclear norm minimization is used as a convex relaxation, which transforms Eq. (20) into a solvable convex optimization problem:

$$\min_{T^*} \left\|T^*\right\|_* \quad \text{s.t} \quad \mathrm{p}_\Omega\left(T^*\right) = \mathrm{p}_\Omega\left(T\right) \tag{22}$$

where is the nuclear norm of the matrix, i.e., the sum of its singular values. The singular value thresholding (SVT) algorithm is used to solve this convex optimization problem, and the iteration is: $\|\cdot\|_*$

$$T^{(k+1)} = D_t\left(T^k + p_\Omega\left(T - T^k\right)\right)$$

where is the intermediate matrix at the k-th iteration, τ is the iteration step size, and convergence is guaranteed when ; to accelerate convergence, is chosen (is an adjustment parameter usually taken in the range 0.1-0.5, and T is the number of snapshots); is the singular value shrinkage operator, defined as: T^k $0 \prec t \prec 2/\left\|p_\Omega\right\|^2$ $t = \beta\sqrt{N_v}/T$ β $D_t(\cdot)$

$$D_t\left(U\Sigma V^H\right) = UD_t\left(\Sigma\right)V^H, \ \left[D_t\left(\Sigma\right)\right]_{i,i} = \max\left(\sigma_i - t, o\right) \tag{24}$$

where is the singular value decomposition of the matrix and is the i-th singular value of the matrix. $U\Sigma V^H$ σ_i

During the SVT iterations, if the singular values at the k-th iteration satisfy , the step size is reasonable; otherwise the step increment is increased until the singular values fall below the threshold. When the convergence condition is satisfied (is the matrix completion convergence threshold, usually taken as), the iteration terminates and the completed covariance matrix is output. $\sigma_i \prec t$ $\left\|T^{(k+1)} - T^{(k)}\right\|F / \left\|T^k\right\|F \prec \varepsilon_0$

ε_0 $10^{-5} - 10^{-3}$ T^*

2.5 DOA Estimation Combined with the Subspace Method

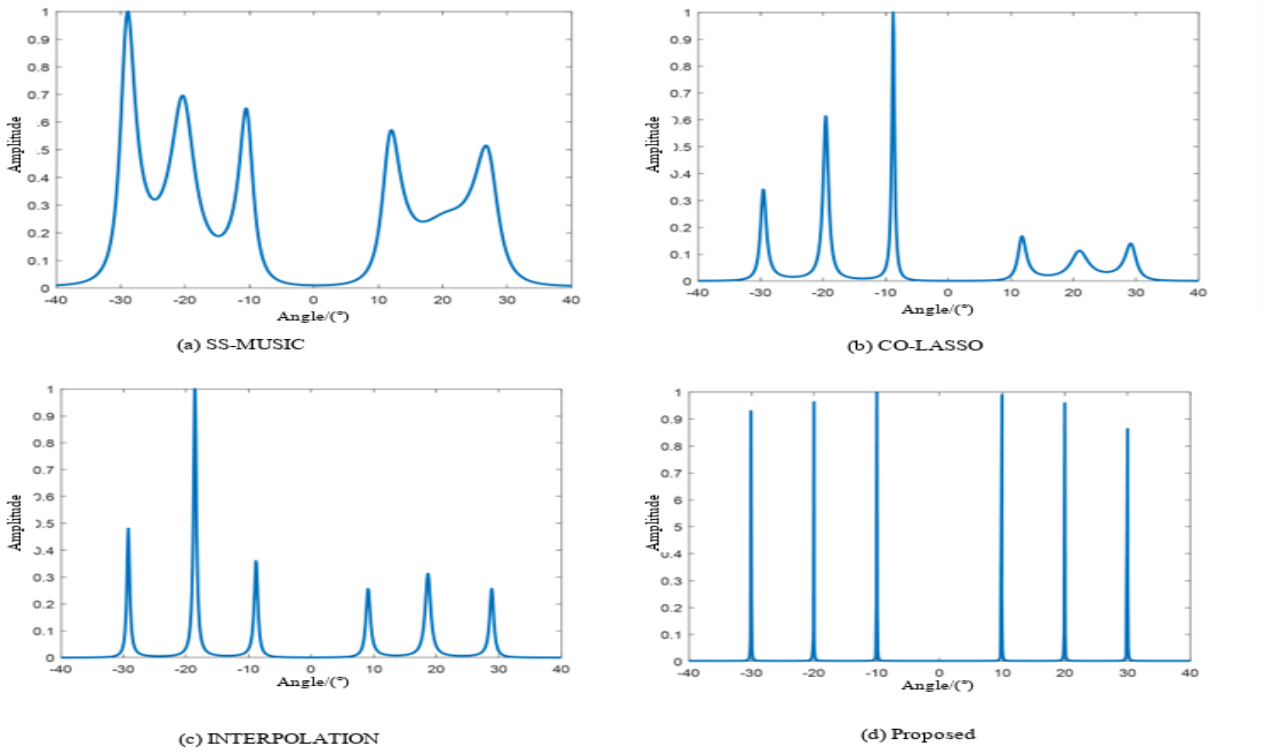
The completed covariance matrix fully incorporates the information of all virtual sensors of the difference co-array, and is eigendecomposed as: T^*

$$T^* = U_s\Sigma_s U_s^H + U_n\Sigma_n U_n^H \tag{25}$$

where and are the diagonal matrices of the signal and noise singular values, respectively; is the basis matrix of the signal subspace, whose column vectors span the signal subspace; and is the basis matrix of the noise subspace, whose column vectors span the noise subspace. The signal subspace and the

International Journal of Science and Engineering Applications
Volume 15-Issue 12, 19 – 27, 2026, ISSN:- 2319 - 7560
DOI: 10.7753/IJSEA1512.1006

noise subspace are mutually orthogonal. Σ_S U_n Σ_S



(a) SS-MUSIC (b) CO-LASSO

(c) INTERPOLATION (d) Proposed

U_n

Based on the orthogonality between the subspaces, the MUSIC algorithm is used to search for spectral peaks, and the spatial spectrum function is constructed as:

$$P_{MUSIC}(\theta)=\frac{1}{\left\|a_v^H(\theta)U_n\right\|^2} \qquad (26)$$

where is the steering vector of the virtual uniform linear array, corresponding to the virtual sensor positions of the difference co-array. The peak positions of the spectrum are the estimated DOAs. $a_v(\theta)$ $\hat{\theta}=\left[\hat{\theta}_1,\hat{\theta}_2,\cdots,\hat{\theta}_K\right]^T$

3 Simulation Results

This section verifies the effectiveness of the proposed algorithm by simulation experiments. The adopted coprime array has sensors, and the number of sensors is . The SNR is defined as

$M=3,N=5$ $2M+N-1=10$。

$$SNR=10\lg\frac{trace(R_S)}{\partial_1^2+\partial_2^2+\cdots+\partial_{2M+N-1}^2} \qquad (27)$$

The root mean square error (RMSE) is defined as

$$RMSE=\sqrt{\frac{1}{JL}\sum_{j=1}^{J}\sum_{l=1}^{L}\left(\hat{\theta}_l(j)-\theta_l\right)^2} \qquad (28)$$

where is the number of Monte Carlo experiments and is the number of signals, J L

θ_l is the true angle of the k-th incident signal, and is its estimate at the j-th trial. l $\hat{\theta}_l(j)$ θ_l j

3.1 Feasibility Analysis

The performance of the proposed algorithm is compared with three classical coprime-array DOA estimation algorithms, namely the SS-MUSIC algorithm, the CO-Lasso algorithm and the interpolation algorithm. Six far-field narrowband signals are uniformly distributed in the range of -30° to 30°, the SNR is , the number of snapshots is L=300, and the noise covariance matrix is . The experimental results of the four algorithms are shown in Fig. 2. $SNR=0dB$

$$R_n=diag\{1,25,31,49,22,34,44,23,13,33\}$$

It can be seen from Fig. 2 that none of the three classical coprime-array DOA estimation algorithms performs satisfactorily: the SS-MUSIC algorithm can only estimate a few sources, the CO-Lasso and interpolation algorithms exhibit large estimation errors for some sources, whereas the proposed

Figure 4 Feasibility Analysis

International Journal of Science and Engineering Applications
Volume 15-Issue 12, 19 – 27, 2026, ISSN:- 2319 - 7560
DOI: 10.7753/IJSEA1512.1006

algorithm can estimate all sources accurately. This is because the three classical algorithms are based on the assumption of uniform white noise and their performance degrades under non-uniform noise, whereas the proposed algorithm effectively suppresses non-uniform noise by matrix reconstruction and completion of the received data covariance.

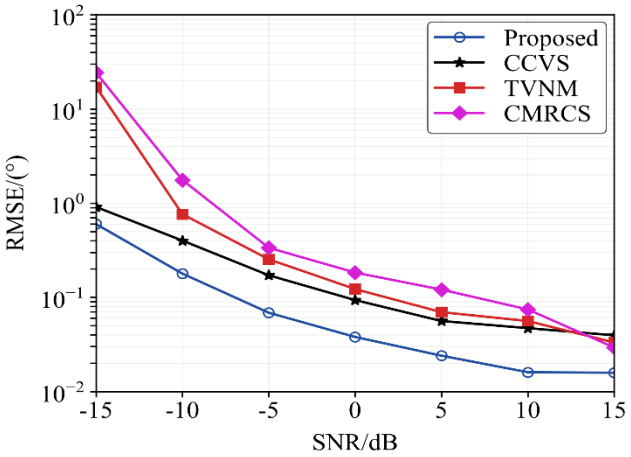


Fig. 5 RMSE versus SNR

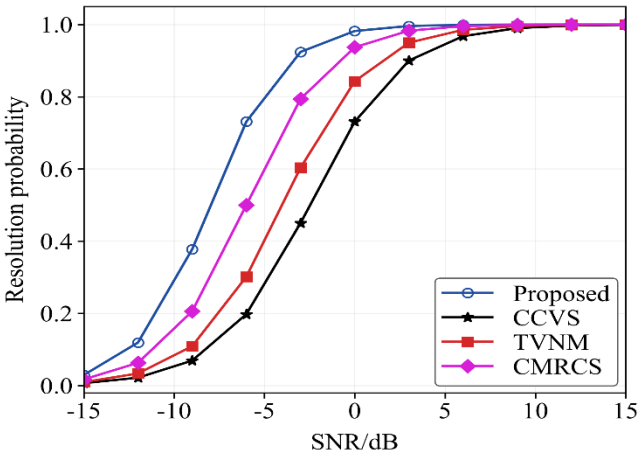


Fig. 6 Resolution probability versus SNR

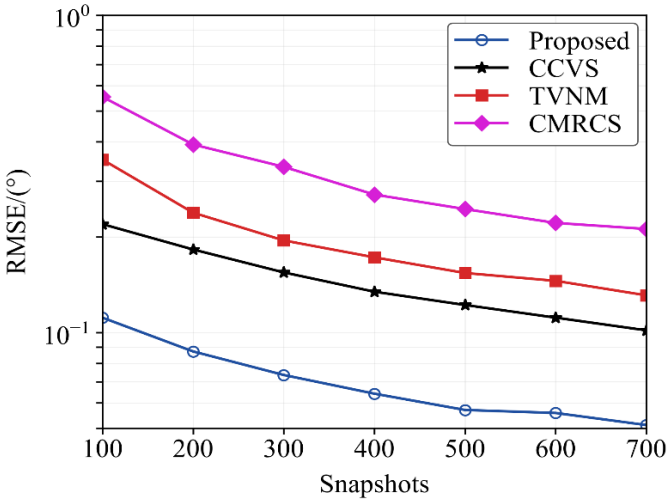


Fig. 7 RMSE versus the number of snapshots

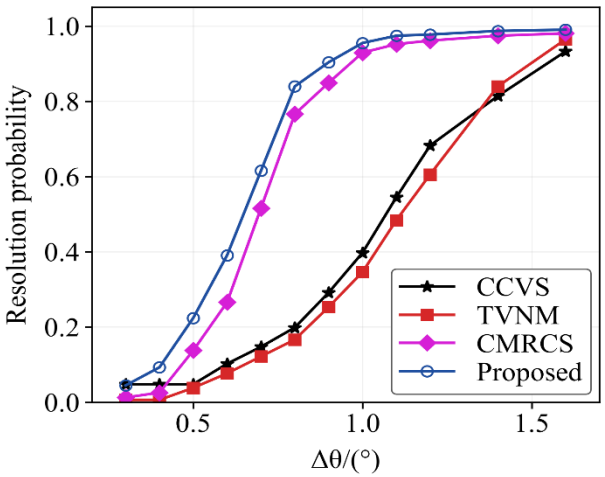


Fig. 8 Resolution probability versus angular separation

3.2 Accuracy Analysis

The performance of the proposed algorithm is compared with three DOA estimation algorithms for coprime arrays under non-uniform noise, namely the CCVS algorithm, the TVNM algorithm and the CMRCS algorithm. Twelve far-field narrowband signals are uniformly distributed in the range of -30° to 30°. Figs. 3 and 4 show the RMSE versus SNR and versus the number of snapshots, respectively. In Fig. 3 the SNR ranges from -10 to 10 dB and 500 Monte Carlo experiments are performed. It can be seen from Fig. 3 that, under the same conditions, the estimation accuracy of all four algorithms improves as the SNR increases, and at high SNR and large snapshot numbers the accuracy of the CCVS and TVNM algorithms is clearly lower than that of the CMRCS algorithm and the proposed algorithm. This is because the CCVS algorithm only uses the continuous virtual sensors of the difference co-array and discards the discontinuous part, which results in a loss of useful information; moreover, both algorithms essentially whiten the noise covariance matrix by averaging the diagonal elements of the covariance matrix, which provides only limited noise suppression. The TVNM algorithm and the proposed algorithm both fully exploit all virtual sensors. The TVNM algorithm constructs a noise-free covariance matrix by removing the diagonal elements of the sample covariance matrix, but this also removes the diagonal elements of the signal covariance matrix and thus causes information loss. The CMRCS algorithm replaces the diagonal elements with the minimum diagonal element, which suppresses non-uniform

noise while preserving useful information; however, when the sample covariance matrix cannot be guaranteed to be accurate, directly replacing the diagonal elements leads to a loss of effective information. In contrast, the proposed method first obtains an accurate covariance matrix by iterative weighting and then replaces the diagonal elements with the minimum diagonal element, which effectively suppresses non-uniform noise while preserving useful information.

3.3 Resolution Probability Analysis

Assume that the true angles of two incident signals are and , and their estimates are and . If both and are smaller than , the two signals are considered to be successfully resolved. Fig. 4 shows the resolution probability versus SNR, where and , the SNR ranges from -10 to 10 dB, and 300 Monte Carlo experiments It can be seen from Fig. 4 that the proposed algorithm has the highest resolution probability. When the SNR increases to 0 dB, the resolution probabilities of the proposed algorithm and the CMRCS algorithm both approach 100%, whereas those of the CCVS and TVNM algorithms remain relatively low. The proposed algorithm therefore has a clear resolution advantage over the CCVS and TVNM algorithms, and exhibits a significant high-resolution advantage at different SNRs.

4 Conclusion

To address the performance decline of underdetermined DOA estimation for coprime arrays under non-uniform noise conditions, this paper proposes a DOA estimation method based on diagonal-masked covariance matrix reconstruction. The method first points out that non-uniform noise only contaminates the diagonal elements of the covariance matrix, while the off-diagonal elements are noise-free and contain all the angle information; theoretical analysis then proves that the diagonal scalars of the reconstructed Toeplitz matrix only shift the eigenvalues overall without altering the noise subspace, thus carrying no angle information. These can be automatically determined with semi-definite constraints and trace minimisation, converging to the ideal value under any noise distribution. Compared with existing methods that replace the minimum diagonal elements, this method eliminates the introduced systematic bias and does not rely on empirical parameters. By combining singular value threshold matrix filling to fully utilise all virtual elements of the difference co-array, simulation results show that this method outperforms comparative algorithms under few snapshots, low SNR, and closely spaced sources, and in high SNR the diagonal-constrained reconstruction algorithm still has irreducible residual failures, confirming the correctness of the theoretical analysis.

References

[1] VAIDYANATHAN P P and PAL P. Sparse sensing with co-prime samplers and arrays[J]. IEEE Transactions on Signal Processing, 2011, 59(2): 573–586.

[2] CHEN Chuan, GUO Yong, and JIA Yong. DOA estimation method based on dual-frequency coprime array[J]. Application Research of Computers, 2017, 34(05): 1459-1462.

[3] SU Yao. DOA Estimation of Coprime Array Based on Compressed Sensing[D]. Xidian University, 2017.

[4] WU Chenxi, ZHANG Min, and WANG Keren. Underdetermined DOA estimation method for coprime array based on matrix completion[J]. Advanced Engineering Sciences, 2017, 49(05): 156-163.

[5] YAN Xuewei. Underdetermined DOA Estimation Based on Coprime Array[D]. Harbin Institute of Technology, 2018.

[6] QIN Si, ZHANG Y D, and AMIN M G. Generalized coprime array configurations for direction-of-arrival estimation[J]. IEEE Transactions on Signal Processing, 2015, 63(6): 1377–1390.

[7] WANG Xiaomeng and WANG Xin. Hole identification and filling in k-times extended Processing, 2019, 67(10): 2693–2706.

[8] SUN Bing, WU Chenxi, and RUAN Huailin. Array diagnosis and DOA estimation for coprime array under sensor failures[J]. Sensors, 2020, 20(9): 2735.

[9] RAZA A, LIU Wei, and SHEN Qing. Thinned coprime array for second-order difference co-array generation with reduced mutual coupling[J]. IEEE Transactions on Signal Processing, 2019, 67(8): 2052–2065.

[10] PAL P and VAIDYANATHAN P P. Coprime sampling and the music algorithm[C]. 2011.

[11] Digital Signal Processing and Signal Processing Education Meeting, Sedona, USA, 2011: 289–294.

[12] ZHANG Y D, AMIN M G, and HIMED B. Sparsity-based DOA estimation using co-prime arrays[C]. 2013 IEEE International Conference on Acoustics, Speech and Signal Processing, Vancouver, Canada, 2013: 3967–3971.

[13] MATVEYEV A L, GERSHMAN A B, and BÖHME J F. On the direction estimation Cramér-Rao bounds in the presence of uncorrelated unknown noise[J]. Circuits, Systems and Signal Processing, 1999, 18(5): 479–487.

[14] GERSHMAN A B, MATVEYEV A L, and BOHME J F. Maximum likelihood estimation of signal power in sensor array in the presence of unknown noise field[J]. IEE Proceedings - Radar, Sonar and Navigation, 1995, 142(5): 218–224.

[15] ESAVENTO M and GERSHMAN A B. Maximum likelihood direction-of-arrival estimation in the presence of unknown nonuniform noise[J]. IEEE Transactions on Signal Processing, 2001, 49(7): 1310–1324.

[16] U Y, HOU C, LIAO G, et al. Direction-of-arrival estimation in the presence of unknown nonuniform noise fields[J]. IEEE Journal of Oceanic Engineering, 2006, 31(2): 504–510.

[17] HEN C E, LORENZELLI F, HUDSON R E, et al. Stochastic maximum-likelihood DOA estimation in the presence of unknown nonuniform noise[J]. IEEE Transactions on Signal Processing, 2008, 56(7): 3038–3044.

[18] IAO Bin, CHAN S C, HUANG Lei, et al. Iterative methods for subspace and DOA estimation in nonuniform noise[J]. IEEE Transactions on Signal Processing, 2016, 64(12): 3008–3020.

[19] FANDIARI M, VOROBYOV S A, ALIBANI S, et al. Non-iterative subspace-based DOA estimation in the presence of nonuniform noise[J]. IEEE Signal Processing Letters, 2019, 26(6): 848–852.

[20] Zhenqing, SHI Zhiping, and HUANG Lei. Covariance sparsity-aware DOA estimation for nonuniform noise[J]. Digital Signal Processing, 2014, 28: 75–81.

[21] E Zhenqing, SHI Zhiping, HUANG Lei, et al. Underdetermined DOA estimation for wideband signals using robust sparse covariance fitting[J]. IEEE Signal Processing Letters, 2015, 22(4): 435–439.

[22] IAN Ye, SHI Hongyin, and XU He. DOA estimation in the presence of unknown non-uniform noise with coprime array[J]. Electronics Letters, 2017, 53(2): 113–115.

[23] WU Chenxi, ZHANG Min, and WANG Keren. Underdetermined DOA estimation in non-uniform noise[J]. Systems Engineering and Electronics, 2018, 40(3): 498-503.

[24] SUN Bing, RUAN Huailin, WU Chenxi, et al. Underdetermined DOA estimation method for coprime array under non-uniform noise[J]. Journal of Electronics & Information Technology, 2021, 43(12): 3687-3694.

[25] IU Ke and ZHANG Y D. Coprime array-based DOA estimation in unknown nonuniform noise environment[J].Digital Signal Processing, 2018, 79: 66–74.