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Dynamic Event-Triggered Prescribed-Time Bipartite Consensus for Parameter-Uncertain MELS

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Abstract: The dynamic event-triggered prescribed-time bipartite consensus problem for multiple Euler-Lagrange systems (MELS) with parameter uncertainties is studied. Firstly, a dynamic event-triggering mechanism is designed to reduce communication frequency and improve the utilization of communication resources. Using the system's linear parameterization, the parameter adaptive update law is designed to estimate and compensate for the unknown parameters. On this basis, an adaptive preset-time bipartite consensus controller is designed by combining a time-base generator (TBG), enabling the system to achieve bipartite consensus control at any given time. Then, by using algebraic graph theory and Lyapunov stability theory, the stability of the system is analyzed in detail, and the sufficient conditions for the system to achieve the prescribed time bipartite consensus are given, and it is strictly proved that the system does not exhibit Zeno behavior. Finally, the effectiveness of the proposed method is verified by numerical simulation examples.

Keywords: multiple Euler-Lagrange systems; dynamic event-triggered control; prescribed-time bipartite consensus; parameter uncertainty; time base generator

1. INTRODUCTION

In recent years, research achievements on cooperative control of multi-agent systems have been widely applied to engineering fields such as unmanned aerial vehicle formation [1], intelligent transportation [2], and smart grid [3]. Consensus control is a fundamental issue in the cooperative control of multi-agent systems [4]-[6]. Most traditional consensus studies focus on cooperative relationships among agents. Nevertheless, cooperation and competition usually coexist in practical applications including robot obstacle avoidance [7] and bidirectional flight of unmanned aerial vehicle formations [8]. Accordingly, considerable progress has been made in the research on bipartite consensus control for multi-agent systems [9]-[12].

The Euler-Lagrange system can characterize complex nonlinear dynamic systems such as spacecraft systems [13], aircrafts [14], and manipulators [15], and it is representative in practical engineering systems. Recently, the bipartite consensus control problem for Euler-Lagrange systems (MELS) has attracted increasing research attention [16]-[17]. Nevertheless, the inertial, centrifugal, and gravitational terms in Euler-Lagrange models often contain unknown parameters in practical engineering systems, and external environmental factors further aggravate such parameter uncertainties. To address this challenge, Ref. [18] investigates the fixed-time leader-following consensus control of MELS with parameter uncertainties under directed communication networks. An adaptive sliding-mode estimator is proposed in Ref. [19] to mitigate the adverse effects induced by parameter uncertainties.

Convergence time serves as a critical indicator for evaluating controller performance. Prescribed-time control enables the system to achieve stability within an arbitrarily preassigned time. By virtue of time-varying functions, Ref. [20] develops a prescribed-time sliding-mode controller to realize prescribed-time bipartite consensus for MELS. Based on prescribed-time observers and prescribed-time trackers, Ref. [21] addresses the prescribed-time leader-following consensus control problem for MELS subject to external disturbances. On the other hand, continuous control strategies consume substantial communication resources. By introducing an event-triggered mechanism, agents communicate and update control inputs only when their state errors exceed predefined thresholds. With the aid of the event-triggered mechanism, Ref. [22] achieves bipartite consensus control for MELS in the presence of external disturbances. In Ref. [23], a dynamic event-triggered control algorithm is constructed to solve the leader-free consensus problem and containment control problem of MELS.

Motivated by the above observations, this paper investigates the prescribed-time bipartite consensus control problem for MELS with parametric uncertainties via a dynamic event-triggered mechanism under network topologies with coexisting cooperation and competition.

2. Preliminaries

2.1 Notation

$\mathbb{R}^{p}$ and $\mathbb{R}^{p\times p}$ denote the p dimensional real column vectors and $p\times p$ dimensional real matrices, respectively. I_p denotes the p dimensional identity matrix. 0_p and 1_p denote the p dimensional column vectors with all entries being zero and one, respectively. N denotes the set of natural numbers. $\|\cdot\|$ denotes the 2-norm of a matrix or the Euclidean norm of a vector. $\otimes$ denotes the Kronecker product. $\sigma_{\max}(P)$ denotes the maximum singular value of matrix $P\in\mathbb{R}^{m\times n}$.

$\lambda_{\min}(Q)$ denotes the minimum eigenvalue of symmetric matrix $Q \in \mathbb{R}^{n\times n}$.

2.2 Algebraic graph theory

We adopt a signed graph. $\mathrm{G}=\{\mathrm{V},E,A\}$ to describe the communication network among agents. Here, $\mathrm{V}=\{v_1,v_2,...,v_N\}$ denotes the node set; $E\subseteq \mathrm{V}\times\mathrm{V}$ denotes the edge set, where $(v_j,v_i)\in E$ means that agent i can receive information from agent j ; $A=[a_{ij}]\in\mathbb{R}^{N\times N}$ denotes the adjacency matrix of G . If $(v_j,v_i)\in E$, then $a_{ij}\neq 0$; otherwise, $a_{ij}=0$ .The sign of a_{ij} reflects the relationship between node v_i and node v_j . If the edge (v_j,v_i) exists, then v_i and v_j are neighboring nodes. A path from node v_i to node v_j is a sequence of edges in the form of $(i,i_{k1}),(i_{k1},i_{k2}),...,(i_{kl},j)$, where i_{kb} represent distinct nodes, $b=1,2,...,j-1,i\neq j$. For the signed graph G , if any two nodes can be connected by a path, then the signed graph G is said to be strongly connected. The Laplacian matrix of the signed graph G is defined as $L=[l_{ij}]\in\mathbb{R}^{N\times N}$, where $l_{ii}=\sum_{j=1,j\neq i}^{N}|a_{ij}|, l_{ij}=-a_{ij}, i\neq j$. For the signed graph G , if the node set V can be divided into two sets V_1 and V_2 satisfying $\mathrm{V}_1\bigcup\mathrm{V}_2=\mathrm{V},\mathrm{V}_1\bigcap\mathrm{V}_2=\varnothing, a_{ij}\geq 0$ for $\forall v_i,v_j\in\mathrm{V}_m, m\in\{1,2\}$, and $a_{ij}\leq 0$ for $\forall v_i\in\mathrm{V}_m, v_j\in\mathrm{V}_{3-m}, m\in\{1,2\}$, then the graph G is called a structurally balanced graph; otherwise, it is called a structurally unbalanced graph.

2.3 Problem Formulation

Consider a multi-Euler-Lagrange system consisting of N agents. The dynamic model of the i -th agent is given by

$$M_i(q_i(t))\ddot{q}_i(t)+C_i(q_i(t),\dot{q}_i(t))\dot{q}_i(t)+g_i(q_i(t))=\tau_i(t) \quad (1)$$

where $q_i(t),\dot{q}_i(t),\ddot{q}_i(t)\in\mathbb{R}^p$ denote the generalized coordinate, generalized velocity, and generalized acceleration vectors, respectively; $M_i(q_i(t))\in\mathbb{R}^{p\times p}$ is the positive-definite inertia matrix; $C_i(q_i(t),\dot{q}_i(t))\in\mathbb{R}^{p\times p}$ denotes the Coriolis-centrifugal matrix; $g_i(q_i(t))\in\mathbb{R}^p$ represents the gravity vector; $\tau_i(t)\in\mathbb{R}^p$ is the control input applied to the system. The system model (1) satisfies the following fundamental properties [23].

Property 1: The matrix $\dot{M}_i(q_i)-2C_i(q_i,\dot{q}_i)$ is skew-symmetric, $x^T[\dot{M}_i(q_i)-2C_i(q_i,\dot{q}_i)]x=0,\forall x\in\mathbb{R}^p$.

Property 2: For any $x,y\in\mathbb{R}^p$, the MELS can be linearly parameterized as $M_i(q_i)x+C_i(q_i,\dot{q}_i)y+g_i(q_i)=Y_i(q_i,\dot{q}_i,x,y)\theta_i$ where $Y_i(q_i,\dot{q}_i,x,y)$ is the regression matrix, and θ_i is the constant parameter vector.

Property 3: For the i -th agent, there exist positive constants $k_{\underline{m}},k_{\bar{m}},k_C,k_g$ such that $k_{\underline{m}}\leq\|M_i(q_i)\|\leq k_{\bar{m}},\|C_i(q_i,\dot{q}_i)\|\leq k_C,\|g_i(q_i)\|\leq k_g$.

The following assumptions and lemmas are used to prove the main results of this paper.

Assumption 1: The signed graph G is strongly connected and structurally balanced.

Assumption 2: Each entry of $M_i(q_i),C_i(q_i,\dot{q}_i),g_i(q_i)$ is differentiable and locally Lipschitz.

Lemma 1 [9]: If the signed graph G is structurally balanced, there exists a diagonal matrix $\mathrm{D}=diag\{\sigma_1,\sigma_2,...,\sigma_N\}$ such that all entries of DAD are non-negative, $\mathrm{DAD}\geqslant 0$, where $\sigma_i\in\{1,-1\}, i\in\mathrm{V}$.

Lemma 2 [24]: Consider the TBG with the following form:

$$\xi(t)=\begin{cases}1-\rho(t)\eta, t\in[0,T_f], \\ 1, \quad t\in(T_f,\infty),\end{cases} \quad (2)$$

where the constant $T_f>0$ is the prescribed-time instant; $\eta=\left[\rho^T(t_0),\rho^T(T_f)\right]^{-T}\lambda\in\mathbb{R}^{2n+2}, \lambda=[1,0,...,0]^T\in\mathbb{R}^{2n+2}, \boldsymbol{\rho}(t)=\left[\rho^T(t),\dot{\rho}^T(t),...,\rho^{(n)^T}(t)\right]^T\in\mathbb{R}^{(n+1)\times(2n+2)}; \rho(t)=\left[t^{2n+1},t^{2n},...,t,1\right]\in\mathbb{R}^{2n+2}$ .The TBG (2) satisfies the following properties: $\xi(t)$ is a continuous non-decreasing function; $\xi(0)=0,\dot{\xi}(0)=0$; $\xi(t)\equiv 1,\dot{\xi}(t)\equiv 0, T_f\leq t<\infty$ .With the aid of TBG (2), construct the gain function $h(t)=\upsilon\dot{\xi}(t)/\left(1-\xi(t)+\delta\right)$, where $\delta>0,\upsilon>0$.

Lemma 3 [24]: Consider the linear time-varying system

$$\dot{p}(t)=-h(t)p(t), p(t)=p_0 \quad (3)$$

where p_0 is the initial value of $p(t)$. The system achieves practical prescribed-time convergence, $p(t)$ converges to $[\delta/(1+\delta)]^{\upsilon}p_0$ for $t\geq T_f>0$, and the positive settling time T_f can be prescribed in advance.

Lemma 4 [23]: Let G be a directed graph consisting of N nodes, and $L\in\mathbb{R}^{N\times N}$ be its associated Laplacian matrix. The following two conclusions hold: If and only if G contains a directed spanning tree, the Laplacian matrix L has a unique zero eigenvalue, and the real parts of all other eigenvalues are positive; If G is strongly connected, there exists a vector $\zeta\triangleq[\zeta_1,\zeta_2,...,\zeta_n]^T\in\mathbb{R}^N$ with $\sum_{i=1}^{n}\zeta_i=1,\zeta_i>0$, such that $\zeta^T L=0$.

Lemma 5 [23]: Suppose that the directed graph induced by the unsigned adjacency matrix $|A|$ is strongly connected. Define matrix $B\triangleq\varXi\bar{L}+\bar{L}\varXi$, where $\varXi\triangleq diag(\zeta_1,...,\zeta_n)$, and ζ is defined in Lemma 4. Then B is the Laplacian matrix corresponding to a certain undirected graph. For any nonzero vector $\vartheta\in\mathbb{R}^N$ satisfying $\vartheta^T\zeta=0$, we have

$$a(\bar{L})\triangleq min\frac{\vartheta^T B\vartheta}{\vartheta^T\varXi\vartheta}>0 \quad (4)$$

Furthermore, one obtains $\vartheta^T B\vartheta\geq a(\bar{L})\vartheta^T\varXi\vartheta\geq a(B)\vartheta^T\vartheta$, where $a(B)=a(\bar{L})\min(\zeta_1,...,\zeta_n)$.

For the MELS (1), the definition of prescribed-time bipartite consensus is given as follows:

Definition 1: For the MELS (1), if the system states satisfy the following relations:

$$\begin{cases} \lim\limits_{t\to T_f}\|q_i(t)-q_j(t)\|=0, & i,j\in V_1 \ or\ i,j\in V_2 \\ \|q_i(t)-q_j(t)\|=0,\ \forall t\ge T_f, & i,j\in V_1 \ or\ i,j\in V_2 \\ \lim\limits_{t\to T_f}\|q_i(t)+q_j(t)\|=0, & i\in V_1, j\in V_2 \\ \|q_i(t)+q_j(t)\|=0, & \forall t\ge T_f,\ i\in V_1, j\in V_2 \end{cases}$$

then the MELS (1) is said to achieve prescribed-time bipartite consensus, where $T_f>0$ is the prescribed time, which can be arbitrarily preset according to practical requirements.

In event-triggered control, avoiding Zeno behavior is a critical issue. The definition of Zeno behavior is presented below.

Definition 2: Within a finite time interval $t\in[t_1,t_2]$ with $0\le t_1<t_2<\infty$, if events are triggered infinitely many times with the event-triggering time sequence $\{t_k^i,\dots,t_\infty^i\}$ (k is a finite positive integer), then the system is said to exhibit Zeno behavior.

3. Main Results

In this section, a dynamic event-triggered prescribed-time bipartite consensus controller is designed for the parameter-uncertain MELS (1), which achieves the control objective given in Definition 1 and excludes Zeno behavior. First, introduce the auxiliary variable $z_i(t)=\sigma_i q_i(t)$, where $\sigma_i=1, i\in V_1; \sigma_i=-1, i\in V_2$. The original system is transformed into a consensus problem under the z_i coordinate. Then, the relative position information $x_i(t)$ and sliding-mode variable $s_i(t)$ are designed as

$$x_i(t)=-\sum_{j=1}^{n}|a_{ij}|(z_i(t)-z_j(t)) \tag{5}$$

$$s_i(t)=\dot z_i(t)-x_i(t)=\dot z_i(t)+\sum_{j=1}^{n}|a_{ij}|(z_i(t)-z_j(t)) \tag{6}$$

The event-triggered mechanism is adopted in this paper. Let t_k^i denote the k -th event-triggering instant of the i -th agent in MELS. The distributed adaptive prescribed-time bipartite consensus controller is designed as follows:

$$\begin{cases} \tau_i(t)=-K_i s_i(t_k^i)+Y_i(t_k^i)\theta_i(t_k^i), t\in[t_k^i,t_{k+1}^i) \\ K_i(t)=k_i+h(t) \\ \dot{\hat\theta}_i(t)=-\Lambda_i Y_i^T(t)s_i(t) \end{cases} \tag{7}$$

where k_i and Λ_i are positive control gains; $\hat\theta_i(t)$ is the estimate of the unknown parameter θ_i; $\dot{\hat\theta}_i(t)=-\Lambda_i Y_i^T(t)s_i(t)$ represents the parameter adaptive update law to compensate for system dynamic parameter uncertainties; $Y_i(t_k^i)$ is the shorthand notation for the regression matrix $Y_i(q_i(t_k^i),\dot q_i(t_k^i),0,\dot x_i(t_k^i))$ defined in Property 2, which satisfies

$$Y_i(q_i(t_k^i),\dot q_i(t_k^i),0,\dot x_i(t_k^i))=C_i(q_i(t_k^i),\dot q_i(t_k^i))\dot x_i(t_k^i)+g_i(q_i(t_k^i))$$

Define the measurement errors $e_i(t)$ and $\varepsilon_i(t)$ as

$$\begin{cases} e_i(t)=s_i(t_k^i)-s_i(t) \\ \varepsilon_i(t)=Y_i(t_k^i)\hat\theta_i(t_k^i)-Y_i(t)\hat\theta_i(t) \end{cases} \tag{8}$$

Define the dynamic variable of the i -th agent as $\chi_i(t)$, whose form is given by

$$\dot\chi_i(t)=-\gamma_i\chi_i(t)-\omega_i\left[\|\varepsilon_i(t)\|+k_i\|e_i(t)\|-\beta_i\|s_i(t)\|\right] \tag{9}$$

where γ_i is a positive constant; ω_i and β_i are nonnegative constants. The initial value of this internal dynamic variable $\chi_i(0)$ is a positive constant. To determine the event-triggering time sequence $\{t_k^i\}$, the inequality for the dynamic event-triggering condition is expressed as:

$$\|\varepsilon_i(t)\|+k_i\|e_i(t)\|-\beta_i\|s_i(t)\|>\chi_i(t) \tag{10}$$

When condition (10) holds, the controller updates the received state information, and the measurement errors are reset to zero. From the dynamic variable (9) and the dynamic event-triggering condition (10), we have

$$\chi_i(t)\ge\chi_i(0)e^{-(\gamma_i+\omega_i)t} \tag{11}$$

Theorem 1: Under Assumption 1 and Assumption 2, consider the MELS (1) with the dynamic event-triggering condition (10). Under the controller (7), if the parameter k_i satisfies

$$k_i>max\mu_i+k_{\bar m}\sigma_{max}(\bar L)+\frac{\left[k_{\bar m}\sigma^2{}_{max}(\bar L)+1\right]^2}{2a(B)}\quad \forall i=1,\dots,n \tag{12}$$

where $\mu_i=(1+2\beta_i+\beta_i\omega_i)/2$, then the MELS (1) achieves prescribed-time bipartite consensus.

Proof: Substitute the controller (7) into MELS (1), from Property 2 and the sliding-mode variable (6), it follows that

$$\begin{aligned} &M_i(q_i(t))\dot s_i(t)+C_i(q_i(t),\dot q_i(t))s_i(t)=-K_i s_i(t_k^i)+ \\ &Y_i(t_k^i)\theta_i(t_k^i)-Y_i(t)\theta_i-M_i(q_i(t))\dot x_i(t) \end{aligned}$$

Rewrite it in vector form:

$$\begin{aligned} &M(q(t))\dot s(t)+C(q(t),\dot q(t))s(t)=-Ks(t_k)+ \\ &Y(t_k)\theta(t_k)-Y(t)\theta(t)-M(q(t))\dot x(t) \end{aligned} \tag{13}$$

By Lemma 4, define the reference variable $\bar z(t)\triangleq\sum_{i=1}^{n}\zeta_i z_i(t)=\left(\zeta^T\otimes I_p\right)z(t)$, where $z(t)$ is the column-stacked vector of $z_i(t)$. Define $\tilde z_i(t)\triangleq z_i(t)-\bar z(t)$, and let $\tilde z(t)$ be the column-stacked vector of $\tilde z_i(t)$. It follows that $\tilde z(t)=z(t)-\mathbf{1}_n\otimes\bar z(t)$. From the above definitions, we obtain

$$\left(\bar L\otimes I_p\right)\tilde z(t)=\left(\bar L\otimes I_p\right)\left(z(t)-\mathbf{1}_n\otimes\bar z(t)\right)=\left(\bar L\otimes I_p\right)z(t)$$

where the property that all row sums of the Laplacian matrix are zero is utilized. Combining the above formula with the relative-position information (5), we have

$$s(t)=\dot z(t)+\left(\bar L\otimes I_p\right)z(t)=\dot z(t)+\left(\bar L\otimes I_p\right)\tilde z(t) \tag{14}$$

Choose the Lyapunov function:

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$$V(t)=\frac{1}{2}s^T(t)M(q(t))s(t)+\frac{1}{2}\tilde{\theta}^T(t)\Lambda^{-1}\tilde{\theta}(t)+$$
$$\tilde{z}^T(t)\left(\Xi\otimes I_p\right)\tilde{z}(t)+\frac{1}{2}\sum_{i=1}^{n}\chi_i^2(t)$$

where Λ is the block-diagonal matrix of $\Lambda_i I_p$; Ξ is obtained from Lemma 5; $\tilde{\theta}(t)$ is the column vector of parameter estimation errors $\tilde{\theta}_i(t)$ defined by $\tilde{\theta}_i(t)=\hat{\theta}_i(t)-\theta_i$. Let $V_1(t)=s^T(t)M(q(t))s(t)/2+\tilde{\theta}^T(t)\Lambda^{-1}\tilde{\theta}(t)/2+\sum_{i=1}^{n}\chi_i^2(t)/2, V_2(t)=\tilde{z}^T(t)\left(\Xi\otimes I_p\right)\tilde{z}(t)$.

Substituting the controller (7) and equation (13), compute the time-derivative of $V_1(t)$:

$$\dot{V}_1(t)=\frac{1}{2}s^T(t)\dot{M}(q(t))s(t)+s^T(t)C(q(t),\dot{q}(t))s(t)-$$
$$s^T(t)Ks(t_k)+s^T(t)Y(t_k)\theta(t_k)-s^T(t)Y(t)\theta(t)-$$
$$s^T(t)M(q(t))\dot{x}(t)-s^T(t)Y(t)\tilde{\theta}(t)+\sum_{i=1}^{n}\dot{\chi}_i(t)\chi_i(t)$$

Combined with measurement error (8) and Property 1, we obtain

$$\dot{V}_1(t)=-s^T(t)Ks(t_k)+\sum_{i=1}^{n}s_i^T(t)\left[\varepsilon_i(t)-K_ie_i(t)\right]-$$
$$s^T(t)M(q(t))\dot{x}(t)+\sum_{i=1}^{n}\dot{\chi}_i(t)\chi_i(t) \tag{15}$$

Where

$$\sum_{i=1}^{n}s_i^T(t)\left[\varepsilon_i(t)-K_ie_i(t)\right]+\sum_{i=1}^{n}\dot{\chi}_i(t)\chi_i(t)$$
$$\le\sum_{i=1}^{n}\left\{\beta_i\|s_i(t)\|^2+\|s_i(t)\|\chi_i(t)-\gamma_i\chi_i^2(t)+\beta_i\omega_i\|s_i(t)\|\chi_i(t)\right\}$$

From the identity $(x-y)^2\ge 0$, we obtain $xy\le(x^2+y^2)/2$. Apply Young's inequality to the term $(1+\beta_i\omega_i)\|s_i(t)\|\chi_i(t)$:

$$(1+\beta_i\omega_i)\|s_i(t)\|\chi_i(t)\le\frac{1+\beta_i\omega_i}{2}\|s_i(t)\|^2+\frac{1+\beta_i\omega_i}{2}\chi_i^2(t) \tag{16}$$

Substitute (16) into the preceding inequality, yielding

$$\sum_{i=1}^{n}s_i^T(t)\left[\varepsilon_i(t)-K_ie_i(t)\right]+\sum_{i=1}^{n}\dot{\chi}_i(t)\chi_i(t)$$
$$\le\sum_{i=1}^{n}\left[\frac{1+2\beta_i+\beta_i\omega_i}{2}\|s_i(t)\|^2-\left(\gamma_i-\frac{1+\beta_i\omega_i}{2}\right)\chi_i^2(t)\right] \tag{17}$$

From $x^Tpy\le\|x\|\|P\|\|y\|=\sigma_{\max}(P)\|x\|\|y\|,\dot{x}(t)=-\left(\bar{L}\otimes I_p\right)\left[s(t)-\left(\bar{L}\otimes I_p\right)\tilde{z}(t)\right]$, and Property 3, it follows that

$$s^T(t)M(q(t))\dot{x}(t)\le k_{\bar{m}}\sigma_{\max}^2(\bar{L})\|s(t)\|\|\tilde{z}(t)\|+$$
$$k_{\bar{m}}\sigma_{\max}(\bar{L})\|s(t)\|^2 \tag{18}$$

Substituting (17) and the above inequality into (15), one obtains

$$\dot{V}_1(t)\le-\left[k_{\min}+h(t)-\max\mu_i-k_{\bar{m}}\sigma_{\max}(\bar{L})\right]\|s_i(t)\|^2+$$
$$k_{\bar{m}}\sigma_{\max}^2(\bar{L})\|s(t)\|\|\tilde{z}(t)\|-\sum_{i=1}^{n}\left(\gamma_i-\frac{1+\beta_i\omega_i}{2}\right)\chi_i^2(t) \tag{19}$$

where $k_{min}=min\{k_1,\ldots,k_n\}$.

Compute the time-derivative of $V_2(t)$

$$\dot{V}_2(t)=2\tilde{z}^T(t)\left(\Xi\otimes I_p\right)\dot{\tilde{z}}(t) \tag{20}$$

From the definitions of $s(t)$ and $\tilde{z}(t)$, the time-derivative of $\tilde{z}(t)$ is given by

$$\dot{\tilde{z}}(t)=\dot{z}(t)-\mathbf{1}_n\otimes\dot{\bar{z}}(t)$$
$$=s(t)-\left(\bar{L}\otimes I_p\right)\tilde{z}(t)-\mathbf{1}_n\otimes\left[\left(\zeta^T\otimes I_p\right)s(t)\right]$$

Note that

$$\left(\Xi\otimes I_p\right)\left\{s(t)-\mathbf{1}_n\otimes\left[\left(\zeta^T\otimes I_p\right)s(t)\right]\right\}=\left[\left(\Xi-\zeta\zeta^T\right)\otimes I_p\right]s(t)$$

Thus equation (20) becomes

$$\dot{V}_2(t)=-\tilde{z}^T(t)\left[\left(\Xi\bar{L}+\bar{L}^T\Xi\right)\otimes I_p\right]\tilde{z}(t)+$$
$$2\tilde{z}^T(t)\left[\left(\Xi-\zeta\zeta^T\right)\otimes I_p\right]s(t) \tag{21}$$

Let $B=\Xi\bar{L}+\bar{L}^T\Xi$, and note that $\left(\zeta^T\otimes I_p\right)\tilde{z}(t)=\mathbf{0}_p$. By Lemma 5,

$$\tilde{z}^T(t)\left[\left(\Xi\bar{L}+\bar{L}^T\Xi\right)\otimes I_p\right]\tilde{z}(t)=$$
$$\tilde{z}^T(t)\left(B\otimes I_p\right)\tilde{z}(t)\le a(B)\|\tilde{z}(t)\|^2 \tag{22}$$

where $\Xi-\zeta\zeta^T$ is diagonally dominant and positive-semidefinite. By the Gersgorin disc theorem, $\sigma_{\max}(\Xi-\zeta\zeta^T)\le 1/2$. Combining with (21) and the above inequality yields

$$\dot{V}_2(t)\le-a(B)\|\tilde{z}(t)\|^2+\|\tilde{z}(t)\|\|s(t)\| \tag{23}$$

Substitute (19) and apply Young's inequality,we obtain:

$$\dot{V}(t)\le-k_0\|s(t)\|^2-\frac{a(B)}{2}\|\tilde{z}(t)\|^2-$$
$$\sum_{i=1}^{n}\left(\gamma_i-\frac{1+\beta_i\omega_i}{2}\right)\chi_i^2(t) \tag{24}$$

Where

$$k_0=k_{min}+h(t)-max\mu_i-k_{\bar{m}}\sigma_{max}(\bar{L})-\frac{\left[k_{\bar{m}}\sigma^2{}_{max}(\bar{L})+1\right]^2}{2a(B)}$$

Clearly, when the control gains k_i satisfy condition (12), k_0 is a positive constant and $\dot{V}(t)$ is negative-definite.

Since $-k_0\|s(t)\|^2\le-h(t)\|s(t)\|^2$, define c_{χ_i} □ $[\gamma_i-(1+\beta_i\omega_i/2)]$. Then equation (24) becomes

$$\dot{V}(t)\le-h(t)\|s(t)\|^2-\frac{a(B)}{2}\|\tilde{z}(t)\|^2-\sum_{i=1}^{n}c_{\chi_i}\chi_i^2(t) \tag{25}$$

From the definition of time-gain in this paper, $1-\xi(t)+\delta\ge\delta>0$, and $\dot{\xi}(t)$ is bounded on $[0,T_f)$. Thus $h(t)$ is upper-bounded over $[0,T_f)$, and there exists a finite constant $\bar{h}$ □ $suph(t)<\infty$. Accordingly,

$$\dot{V}(t) \le h(t)\left[\|s(t)\|^2 - \frac{a(B)}{2\bar{h}}\|\tilde{z}(t)\|^2 - \sum_{i=1}^{n}\frac{c_{\chi_i}}{\bar{h}}\chi_i^2(t)\right]$$

Set $\alpha_0 \triangleq min\{1, a(B)/(2\bar{h}), c_{\chi_1}/\bar{h}, ..., c_{\chi_n}/\bar{h}\} > 0$, then

$$\dot{V}(t) \le -\alpha_0 h(t)\left(\|s(t)\|^2 + \|\tilde{z}(t)\|^2 + \sum_{i=1}^{n}\chi_i^2(t)\right) \tag{26}$$

Because $M_i(q_i) > 0$ is bounded and $\Xi > 0$, there exists a constant $c_1 > 0$ such that

$$V(t) \ge c_1\left(\|s(t)\|^2 + \|\tilde{z}(t)\|^2 + \sum_{i=1}^{n}\chi_i^2(t)\right)$$

Substituting into (26) yields

$$\dot{V}(t) \le -\frac{\alpha_0}{c_1}h(t)V(t)$$

Let $\alpha \triangleq \alpha_0/c_1 > 0$. We obtain $\dot{V}(t) \le -\alpha h(t)V(t), \forall t \in [0, T_f)$. From Lemma 3, the Lyapunov function $V(t)$ converges to an arbitrarily small neighborhood within the prescribed time T_f.

$$V(t) \le \left(\frac{\delta}{1+\delta}\right)^{\alpha} V(0), \forall t > T_f$$

Further, by the equivalence between $V(t)$ and $\|s(t)\|^2 + \|\tilde{z}(t)\|^2 + \|\chi(t)\|^2$, we have $\tilde{z}(t) \to 0, s(t) \to 0, \chi(t) \to 0, \forall t > T_f$. Consequently, $z_i(t) \to \bar{z}(t), q_i(t) \to \sigma_i\bar{z}(t), \forall i = 1, ..., n$. which means that the MELS achieves bipartite consensus within the prescribed time T_f.

Theorem 2: Consider the MELS (1). Suppose Assumptions 1 and 2 hold. Under the controller (7) and the dynamic event-triggered condition (10), the MELS (1) achieves prescribed-time bipartite consensus free of Zeno behavior; that is, the time interval between any two consecutive triggering instants is strictly positive.

Proof: Define $f_i(t) = \|\varepsilon_i(t)\| + k_i\|e_i(t)\|, \forall i = 1, ..., n$. The time-derivative of $k_i\|e_i(t)\|$

$$\frac{d}{dt}k_i\|e_i(t)\| = k_i\frac{e_i^T(t)\dot{e}_i(t)}{e_i(t)} \le k_i\|\dot{e}_i(t)\| \le k_i\|\dot{s}_i(t)\| \tag{27}$$

From the preceding stability analysis, $s_i(t), \dot{s}_i(t), \dot{z}_i(t), \hat{\theta}_i(t)$ are all bounded. Let $B_e > 0$ be a positive upper bound of $\dot{s}_i(t)$. It follows that

$$\frac{d}{dt}k_i\|e_i(t)\| \le k_i\|\dot{s}_i(t)\| \le k_i B_e \tag{28}$$

Similarly, the time-derivative of $\|\varepsilon_i(t)\|$ satisfies

$$\frac{d}{dt}\|\varepsilon_i(t)\| = \|\dot{\varepsilon}_i(t)\| \le \|\dot{Y}_i(t)\hat{\theta}_i(t)\| + \|Y_i(t)\dot{\hat{\theta}}_i(t)\| \tag{29}$$

By virtue of Controller (7), Assumption 2 and Property 2, $\dot{\hat{\theta}}_i(t), Y_i(t)$ and $\dot{Y}_i(t)$ are bounded. Let $B_\varepsilon > 0$ denote a positive upper bound of $\|\dot{Y}_i(t)\hat{\theta}_i(t)\| + \|Y_i(t)\dot{\hat{\theta}}_i(t)\|$. Then

$$\frac{d}{dt}\|\varepsilon_i(t)\| \le B_\varepsilon \tag{30}$$

Combining (28) and (30),

$$\dot{f}_i(t) = \frac{d}{dt}\|\varepsilon_i(t)\| + \frac{d}{dt}k_i\|e_i(t)\| \le B_\varepsilon + k_i B_e$$

Obviously, $f_i(t) \le \chi_i(t)$ is a sufficient condition for the dynamic triggering condition (10) to hold. For any $t \in [t_k^i, t_{k+1}^i)$

$$f_i(t) = \int_{t_k^i}^{t}\dot{f}_i(\tau)d\tau \le (B_\varepsilon + k_i B_e)(t - t_k^i) \tag{31}$$

Since $e_i(t_k^i)$ and $\varepsilon_i(t_k^i)$ are reset to zero at each triggering instant t_k^i, together with (11), a sufficient condition for $f_i(t) \le \chi_i(t)$ is given by

$$(B_\varepsilon + k_i B_e)(t - t_k^i) \le \chi_i(0)e^{-(\gamma_i + \omega_i)t} \tag{32}$$

Note that the next triggering instant t_{k+1}^i determined by the above condition occurs earlier than the next triggering instant t_{k+1}^i determined by triggering condition (10). Therefore, if Zeno behavior is excluded under the above condition, Zeno behavior can also be excluded under triggering condition (10). Let ι denote the minimal inter-event interval between two consecutive triggering instants t_k^i and t_{k+1}^i. Then l satisfies the following equation:

$$(B_\varepsilon + k_i B_e)l = \chi_i(0)e^{-(\gamma_i + \omega_i)(\iota + t_k^i)} \tag{33}$$

Since the solution to the above equation is always positive over any finite time interval, Zeno behavior is excluded.

4. Numerical Simulations

In this section, simulation results are provided to verify that the designed controller can achieve dynamic event-triggered prescribed-time bipartite consensus control for MELS. Specifically, we consider an MELS consisting of eight 2-degree-of-freedom (2-DOF) manipulators, where each manipulator is regarded as an agent. The dynamic model of each manipulator is given by Equation (1), with $i = 1,2,3,4,5,6,7,8$.

$$M(q) = \begin{bmatrix} u_1 + 2u_2 cosq_2 & u_3 + u_2 cosq_2 \\ u_3 + u_2 cosq_2 & u_3 \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} -u_2\dot{q}_2 sinq_2 & -u_2(\dot{q}_1 + \dot{q}_2)sinq_2 \\ u_2\dot{q}_1 sinq_2 & 0 \end{bmatrix},$$

$$g(q) = \begin{bmatrix} u_4 g\dot{q}_2 cosq_1 + u_5 g cos(q_1 + q_2) \\ u_5 g cos(q_1 + q_2) \end{bmatrix},$$

where $g = 9.8m/s^2$. The parameters are given by $u_1 = m_1r_1^2 + m_2(l_1^2 + r_1^2) + J_1 + J_2, u_2 = m_2 l_1 r_2, u_3 = m_2 r_2^2 + J_2, u_4 = m_1 r_1 + m_2 l_1,$ $u_5 = m_2 l_2$ .The physical parameters are $m_1 = 0.92, m_2 = 1.4, l_1 =$

$1.8, l_2 = 0.42$, $J_1 = m_1 l_1^2/3, J_2 = m_2 l_2^2/3, r_1 = l_1/2, r_2 = l_2/2$ .The parameters of Controller (7) are chosen as $k_i = 6.0, \Lambda_i = 0.5$, and the prescribed-time T_f is set to 6s.For triggering condition (10), the parameters are γ_i=0.6, β_i=2.0, ω_i=0.5, $\chi_i(0)$=1.8 .The initial states $q(0) = \left[q_{10}^T, q_{20}^T\right]^T$ and $\dot{q}(0) = \left[\dot{q}_{10}^T, \dot{q}_{20}^T\right]^T$ of the eight manipulators are set to random values.Furthermore, the manipulators are divided into two subgroups $V_1 = \{1,7\}$ and $V_2 = \{2,3,4,5,6,8\}$. The communication topology graph is depicted in Figure 1.

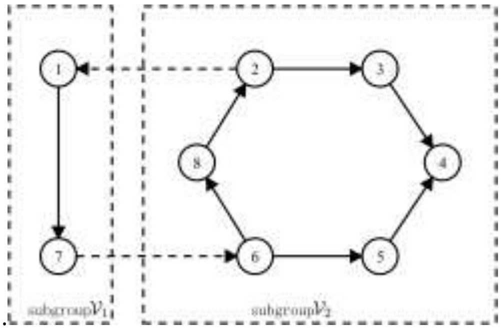

Figure 1 Communication topology

The time-varying curves of the generalized positions and generalized velocities for MELS are shown in Figures 2 and 3.

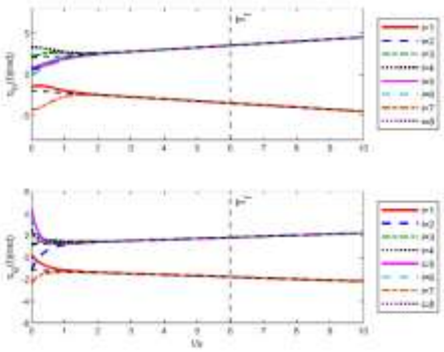

Figure 2 The trajectory change of generalized position in MELS

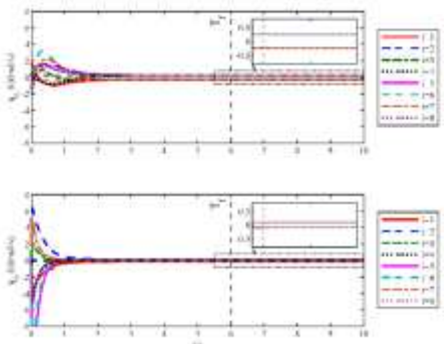

Figure 3 The trajectory change of generalized velocity in MELS

It can be seen from Figures 2 and 3 that under the designed controller, the position and velocity states of all agents converge within the prescribed time, which indicates that the system achieves prescribed-time bipartite consensus.

The triggering counts of each agent under the static event-triggered and dynamic event-triggered mechanisms are depicted in Figure 3.

It can be observed from Figure 3 that the triggering counts of each agent under the dynamic event-triggered mechanism are significantly fewer than those under the static event-triggered mechanism. The proposed dynamic event-triggered mechanism can greatly reduce the triggering frequency of each agent. Therefore, the designed controller can effectively

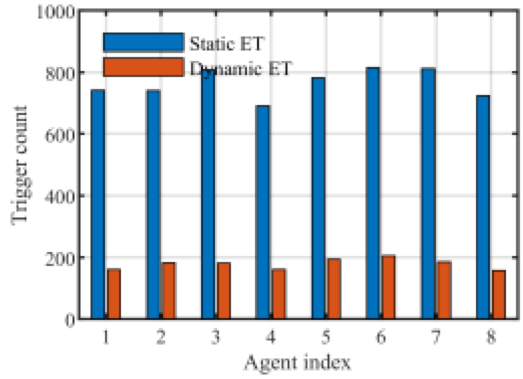

Figure 4 The number of triggers of each agent under different trigger mechanisms

save communication resources while realizing prescribed-time bipartite consensus control.

5. CONCLUSION

This paper investigates the dynamic event-triggered prescribed-time bipartite consensus control problem for MELS with parameter uncertainties. For MELS with parameter uncertainties under structurally balanced graphs, a distributed prescribed-time controller is designed together with the dynamic event-triggered control scheme. It guarantees the achievement of prescribed-time bipartite consensus while saving communication resources. Meanwhile, sufficient conditions ensuring system performance are derived via theoretical analysis, and Zeno behavior is excluded. Finally, simulation experiments are carried out to verify the feasibility of the theoretical results.

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