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Application of Matrices in Human's Life

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Abstract: In this paper, applications of matrices in human's life may be presented by being used the basic concepts of matrices, i.e., addition and multiplication of two matrices, and then, being obtained the determinant of a matrix and the corresponding eigen-values and eigen-vectors. The inverse of a given matrix may be obtained by Gauss's Elimination and Gauss-Jordan. It is essential for this paper.

Keywords: Markov Process, Stochastic Matrix, Gauss Elimination, Gauss-Jordan, eigen-values and eigen-vectors.

1. INTRODUCTION

This paper contains three parts. Firstly, the basic concepts, i.e., definition of a matrix, operations of matrices, classes of real square matrices, will be introduced.

In second portion, matrix eigen values problems containing eigen-values and eigen-vectors, i.e., in Economics, in Engineering problems. Eigen values may be obtained with determinant of $(A - \lambda I)$ is equal to zero and then, the corresponding eigen-vector may be obtained.

Finally, the problem of human's life, Deformation of circular to ellipse, Mixing problem including two tanks, Electrical network problems will be solved with applications of matrices eigen-value problems.

2. TYPESET TEXT

2.1 Definition of a Matrix

A matrix is a rectangular array of numbers (or functions) enclosed in brackets. These numbers function are called entries or elements of the matrix.

A matrix can be denoted by capital boldface letters A, B, C and so on or by writing the general entry in brackets; thus **A**=[a_{jk}], and so on. By an $m \times n$ matrix, we mean a matrix with m rows and n columns. Thus an $m \times n$ matrix is of the form

$$A = [a_{jk}] = \begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$$

A matrix that is not square is called a rectangle matrix. A matrix is a rectangular array of numbers (or functions) enclosed in brackets. These numbers function are called entries or elements of the matrix.

In the double-subscript notation for the entries, the first subscript always denotes the row and the second the column in which the given entry stands. Thus a_{23} is the entry in the second row and third column.

If $m = n$, we call A an $n \times n$ square matrix. Then its diagonal containing the entries $a_{11}, a_{22}, \dots, a_{nn}$ is called the main diagonal or principal diagonal of A. A matrix that is not square is called a rectangle matrix.

2.2 Inverse of a Matrix

The inverse of a matrix may be obtained by Gauss-Jordan Elimination in the form

$$A\,A^{-1} = A^{-1}A = I$$

2.3 Matrix Eigenvalue Problem

The matrix eigen-value problem may be solved as following procedure.

$$AX = \lambda X \to (A - \lambda I)X = 0 \;\&\; |A - \lambda I| = 0$$

2.4 Vectors in terms of Matrices

If a matrix has only one row, then the matrix is said to be a row vector and a matrix with only one column is known as a column vector. In both cases, its entries are vector components and denote the vector by $a = [a_j]$.

2.5 Definition of Matrix Addition

Addition is definition only for matrices A= $[a_{jk}]$ *and* B= $[b_{jk}]$ of the same size; their sum, written A $+ B$, is then obtained by adding the corresponding entries. Matrices of different sizes cannot be added.

2.6 Definition of Scalar multiplication

The product of any matrix A, whose dimension is $m \times n$, i.e., A has m number of rows and n number of columns. In notation, A = $[a_{jk}]$, where j = 1, 2, 3, . . ., m and k = 1, 2, 3, . . ., n, and any scalar c may be written by cA. The product of a matrix and any scalar is also another matrix of same dimension as given matrix. So, cA is also an $m \times n$ matrix.

c A= $[ca_{jk}]$ obtained by multiplying each entry in A by c.

2.7 Properties of matrices

Addition of matrices which have same dimensions obey commutative, associative, identity, inverse properties. In symbol,

(a) $A + B = B + A$

(b) $(U + V) + W = U + (V + W)$

(c) $0 + A = A$

(d) $A + (-A) = 0$

Both multiplication of a scalar with the sum of two or more matrices and multiplication of a matrix with the sum of two or more scalars obey the distributive law. Moreover, scalar multiplication of the matrix, it is obtained by the scalar multiplication of a given matrix with a scalar, with another scalar obey the associative law. Lastly, scalar multiplication obey the identity property. These are

(a) $c(A + B) = cA + cB$

(b) $(c + k) A = cA + k A$

(c) $c(kA) = (ck)A$

(d) I A = A

2.8 Nodal Incidence Matrices

(e) In a Nodal Incidence Matrix, its entries may be written as

(f) $a_{jk} = \begin{cases} +1, & \text{if branch } k \text{ leaves node } j \\ -1 & \text{if branch } k \text{ enters node } j \\ 0, & \text{if branch } k \text{ does not touch node } j \end{cases}$

2. 9 Transposition of a matrix

Consider a matrix $A = [a_{jk}]$is of the form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \dots & a_{1m} \\ a_{21} & a_{22} & a_{23} \dots & a_{2m} \\ a_{31} & a_{32} & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix};$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \dots & a_{n1} \\ a_{12} & a_{22} & a_{32} \dots & a_{n2} \\ a_{13} & a_{23} & \cdots & a_{n3} \\ \vdots & \vdots & \vdots & \vdots \\ a_{1m} & a_{2m} & \cdots & a_{nm} \end{bmatrix}$$

Although A is an $n \times m$ matrix $A^T$is now a $m \times n$ matrix. Hence a matrix and its transport matrix may not have the same dimension. But a square matrix and its transport have same dimension.

2.10 Classes of Real Square Matrices

(Symmetric, Skew-symmetric, and Orthogonal)

Definitions

Consider a real square matrix A = $[a_{jk}]$. Then,

(a) $A^T = A$, i.e., $a_{kj} = a_{jk}$. Then, A is symmetric.

(b) $A^T = -A$,i.e., $a_{kj} = -a_{jk \rightarrow}$ A is skew-symmetric.

(c) $A^T = A^{-1}$,i.e., $A^T = A^{-1}$ →A is orthogonal matrix.

3. MATRIX EIGEN VALUE PROBLEM

3.1 Solution of Linear Systems:

Fundamental Theorem for linear systems

3.1.1Existence

A linear system of m equations in n unknowns $x_1, x_2, x_3, \dots, x_n$ may be

$$x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2 \quad (1)$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_n$$

This may be written by $AX = b$, where

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix},$$

$$\tilde{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} & | & b_1 \\ \vdots & \vdots & \vdots & | & b_2 \\ a_{m1} & \cdots & a_{mn} & | & b_n \end{bmatrix},$$

$$X = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}, \; b = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix},$$

The given linear system is consistent, that is, it has solutions, if and only if the coefficient matrix A and the augmented $\tilde{A}$ have the same rank, say p, i.e., rank of A = rank of$\tilde{A} \leq n$.

3.1.2 Uniqueness

The system (1) has precisely one solution if and only if A and $\tilde{A}$ have the common rank r and r = n. Rank of A = Rank of $\tilde{A}$ = r = n.

3.1.3 Infinitely many solution

The system (1) has infinitely many solutions if and only if A and $\tilde{A}$ have the common rank r, where $r < n$.

rank of A = rank of $\tilde{A} < n$.

3.2 Cramer's Rule

$$x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n = b_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n = b_n$$

Then, $x_1 = \frac{D_1}{D}$, $x_2 = \frac{D_2}{D}, \dots, x_n = \frac{D_n}{D}$, where

$$\det A = D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \cdots & a_{1m} \\ a_{21} & a_{22} & a_{23} \cdots & a_{2m} \\ a_{31} & a_{32} & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{vmatrix}$$

$$.D_1 = \begin{vmatrix} b_1 & a_{12} & a_{13} \cdots & a_{1m} \\ b_2 & a_{22} & a_{23} \cdots & a_{2m} \\ b_3 & a_{32} & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ b_n & a_{n2} & \cdots & a_{nm} \end{vmatrix}, D_2 = \begin{vmatrix} a_{11} & b_1 & a_{13} \cdots & a_{1m} \\ a_{21} & b_2 & a_{23} \cdots & a_{2m} \\ a_{31} & b_3 & \cdots & a_{3m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & b_n & \cdots & a_{nm} \end{vmatrix}$$

3.3 Applications of Matrices

(*EIGEN-VALUED PROBLEMS IN HUMAN'S LIFE*)

3.3.1 Matrix Time Vector, Weight Watching.

Suppose that in a weight-watching program, a person of 185 pounds burns 350 calories per hour in walking (3 miles per hour), 500 calories per hour in bicycling (13 miles per hour) and 950 calories per hour in jogging (5.5 miles per hour). Bill, weighing 185 pounds, plans to exercise according to the matrix shown. Verify the calculation.

(W= Walking, B= Bicycling, J= Jogging).

$$\begin{cases} \text{Monday} \\ \text{Wednesday} \\ \text{Friday} \\ \text{Saturday} \end{cases} \begin{bmatrix} w & b & j \\ 1.0 & 0 & 0.5 \\ 1.0 & 1.0 & 0.5 \\ 1.5 & 0 & 0.5 \\ 2.0 & 1.5 & 1.0 \end{bmatrix} \begin{bmatrix} 350 \\ 500 \\ 950 \end{bmatrix} = \begin{bmatrix} 825 \\ 1325 \\ 1000 \\ 2400 \end{bmatrix} \begin{matrix} \text{Monday} \\ \text{Wednesday} \\ \text{Friday} \\ \text{Saturday} \end{matrix}$$

Total calories lost = 5500 or 1.6 lb per week.

3.3.2 Markov Process with Stochastic Matrix

Suppose that the 2004 state of land use in a city of 60 mi^2 of built-up area is Commercially Used, C 25%, Industrially Used, I 20% and Residentially Used, R 55%. Apply Markov Process to be determined the probabilities of state of land in Commercially Used, Industrially Used and Residentially Used in 2009, 2014 and 2019.Assume that the transition probabilities for 5-year intervals are given by the Stochastic Matrix A.

$$A = \begin{bmatrix} From\ C & From\ I & From\ R \\ 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix} \begin{matrix} To\ C \\ To\ I \\ To\ R \end{matrix}$$

A is a stochastic matrix, that is, a square matrix with all entries nonnegative and all column sums equal to 1. Our example concerns a Markov process, that is, a process for which the probability of entering a certain state depends only on the last state occupied (and the matrix A), not on any earlier state.

Solution: Markov Process with Stochastic Matrix

Let the column vector x denote the 2004 state. Then,

$$y^T = x^T A, z^T = y^T A, u^T = z^T A,$$

$$A = \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix},$$

$x = \begin{bmatrix} 25 \\ 20 \\ 55 \end{bmatrix}$ and $x^T = \begin{bmatrix} 25 \\ 20 \\ 55 \end{bmatrix}^T = [25 \quad 20 \quad 55]$.$y^T, z^T, u^T$must be calculated for required solutions.

Let y denote the 2009 state. Then,

$$y^T = x^T A = [25 \quad 20 \quad 55] \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix}$$

$$y^T = [19.5 \quad 34 \quad 46.5]$$

Let z denote the 2014 state. Then,

$$z^T = y^T A = [19.5 \quad 34 \quad 46.5] \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix}$$

$$z^T = [17.05 \quad 43.8 \quad 39.15]$$

Let u denote the 2019 state.

$$u^T = z^T A = [17.05 \quad 43.8 \quad 39.15] \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0.2 & 0.9 & 0.2 \\ 0.1 & 0 & 0.8 \end{bmatrix}$$

$$u^T = [16.315 \quad 50.660 \quad 33.025]$$

In 2009, the commercial area will be 19.5% (11.7 mi^2), the industrial 34% (20.4 mi^2) and the residential 46.5% (27.9 mi^2).

For 2014, the commercial area will be 17.05%(10.23 mi^2), the industrial 43.8% (26.28 mi^2)and the residential 39.15% (23.49 mi^2).

The next 5 year after 2014, the commercial area will be16.315% (9.789 mi^2), the industrial 50.660 % (30.396 mi^2) and the residential area33.025% (19.815 mi^2).

3.3.3 Gauss- Jordan Elimination

Solve the system of equations

$$2x + y + z = 4$$

$$x + 2y + z = 8$$

$$2 + y + 2z = 4 .$$

Firstly write the given system of equations in the form $AX = B$. Then find $A^{-1}$by being used the Gauss-Jordan's Elimination method.

Solution:

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 4 \end{bmatrix} \to AX = B$$

$$X = A^{-1}B, \text{where } A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 8 \\ 4 \end{bmatrix}$$

Process: Expand A | I.

Start scaling and adding rows to get I | A^{-1}

$A|I =$

$$\begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2}R_1} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_{21}(-1)\&R_{31}(-1)} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{-1}{2} & 1 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-1}{2} & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{3}{2} & \frac{1}{2} & \frac{-1}{2} & 1 & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-1}{2} & 0 & 1 \end{bmatrix} \xrightarrow{R_2\left(\frac{2}{3}\right)} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-1}{2} & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & \frac{1}{2} & \frac{3}{2} & \frac{-1}{2} & 0 & 1 \end{bmatrix} \xrightarrow{R_{32}\left(\frac{-1}{2}\right)} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & \frac{4}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & \frac{4}{3} & \frac{-1}{3} & \frac{-1}{3} & 1 \end{bmatrix} \xrightarrow{R_3\left(\frac{3}{4}\right)} \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{-1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix}$$

$$\xrightarrow{R_{23}\left(\frac{-1}{3}\right)\&R_{13}\left(\frac{-1}{2}\right)} \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{5}{8} & \frac{1}{8} & \frac{-3}{8} \\ 0 & 1 & 0 & \frac{-1}{4} & \frac{3}{4} & \frac{-1}{4} \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{5}{8} & \frac{1}{8} & \frac{-3}{8} \\ 0 & 1 & 0 & \frac{-1}{4} & \frac{3}{4} & \frac{-1}{4} \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix} \xrightarrow{R_{12}\left(\frac{-1}{2}\right)} \begin{bmatrix} 1 & 0 & 0 & \frac{6}{8} & \frac{2}{8} & \frac{-2}{8} \\ 0 & 1 & 0 & \frac{-1}{4} & \frac{3}{4} & \frac{-1}{4} \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix}$$

$$I|A^{-1} = \begin{bmatrix} 1 & 0 & 0 & \frac{6}{8} & \frac{2}{8} & \frac{-2}{8} \\ 0 & 1 & 0 & \frac{-1}{4} & \frac{3}{4} & \frac{-1}{4} \\ 0 & 0 & 1 & \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix} \to A^{-1} = \begin{bmatrix} \frac{3}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{3}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix}$$

$$X = A^{-1}B = \begin{bmatrix} \frac{3}{4} & \frac{-1}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{3}{4} & \frac{-1}{4} \\ \frac{-1}{4} & \frac{-1}{4} & \frac{3}{4} \end{bmatrix}\begin{bmatrix} 8 \\ 8 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix} \to X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$$

3.3.4

Mixing problem involving two tanks

Tank T_1contains initially 100 gal of pure water. Tank T_2 contains initially 100 gal of water in which 150 lb of fertilizer are dissolved. Liquid circulates through the tanks at a constant rate of 2gal/min and the mixture is kept uniform by stirring. Find the amounts of fertilizer $y_1(t)$ $and y_2(t)$ in T_1 and T_2 respectively, where t is time.

Solution:

Let $y_1(t)$ =amount of fertilizer in tank T_1 and

$y_2(t)$ = amount of fertilizer in tank T_2.

Hence, the time rate of change of $y_1(t) = y_1'(t)$ and the time rate of change of $y_2(t) = y_2'(t)$.

Tank.1: (100) gal of water contains y_1 lb of dissolved fertilizer.

(2) gal of water contains $\frac{2}{100}y_1$lb of dissolved fertilizer.

Tank.2: (100) gal of water contains $y_2$lb of dissolved fertilizer.

(2) gal of water contains $\frac{2}{100}y_2$lb of dissolved fertilizer.

In tank1,$y_1'(t) = \frac{Inflow}{min} - \frac{outflow}{min} \to y_1' = \frac{-2}{100}y_1 + \frac{2}{100}y_2$

In tank2,$y_2'(t) = \frac{Inflow}{min} - \frac{outflow}{min} \to y_2' = \frac{2}{100}y_1 - \frac{2}{100}y_2$

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} \frac{-2}{100} & \frac{2}{100} \\ \frac{2}{100} & \frac{-2}{100} \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \to y' = A\,y$$

Let $y = xe^{\lambda t} \to y' = \lambda xe^{\lambda t} = \lambda y = Ay$

$\lambda xe^{\lambda t} = Axe^{\lambda t} \to A\boldsymbol{x} = \lambda\boldsymbol{x}$

$(A - \lambda \mathrm{I})\boldsymbol{x} = 0 \to \det(A - \lambda I) = 0$

$$\begin{vmatrix} -0.02 - \lambda & 0.02 \\ 0.02 & -0.02 - \lambda \end{vmatrix} = 0 \to \lambda = 0\,, \lambda = -0.04$$

$[A - \lambda I]\boldsymbol{x} = 0$

$$\begin{bmatrix} (-0.02 - \lambda) & 0.02 \\ 0.02 & (-0.02 - \lambda) \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$(-0.02 - \lambda)x_1 + 0.02x_2 = 0$

$0.02x_1 + (-0.02 - \lambda)x_2 = 0$

For $\lambda = 0\,, x_1 = x_2 = 1 \to \underline{x_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

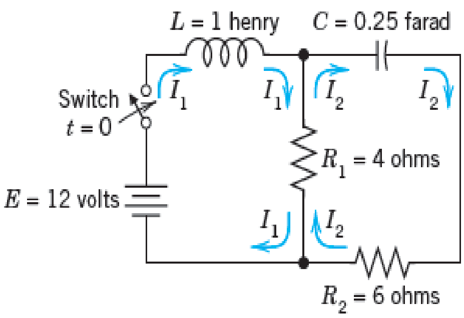
For $\lambda = -0.04$, $x_1 = 1, x_2 = -1 \rightarrow \underline{x_2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$y = c_1 x_1 e^{\lambda_1(t)} + c_2 x_2 e^{\lambda_2(t)}$,

$y_1(0) = 0 , y_2(0) = 150 \rightarrow c_1 = 75 , c_2 = -75$

$y_1 = 75 - 75e^{-0.04t}$ and $y_2 = 75 + 75\, e^{-0.04t}$

3.3.5 Electrical Network



Find the current $I_1(t)$ and $I_2(t)$ in the network in figure. Assume all the currents and charge to be zero at t=0, the instant when the switch is closed.

Solution: Electromotive force,

$E(t) = E_l + E_R + E_c, E_l = l\, I', E_R = R\, I, E_c = \frac{1}{c}\int I\,(t)dt$

For the left loop yield

$$E = E_l + E_{R_1}$$

$$12 = l\, I_1' + R_1(I_1 - I_2) \rightarrow 12 = (1)I_1' + (4)(I_1 - I_2)$$

$$I_1' = -4\, I_1 + 4\, I_2 + 12 \qquad (1)$$

For the right loop yield,

$E = E_{R_1} + E_{R_2} + E_c \rightarrow 0 = R_1\,(I_2 - I_1) + R_2 I_2 + \frac{1}{c}\int I_2(t)dt$

$$0 = 4\,(I_2 - I_1) + 6I_2 + \frac{1}{0.25}\int I_2(t)dt$$

$$0 = 4I_2 - 4I_1 + 6I_2 + 4\int I_2(t)dt$$

Differentiate both sides,

$$0 = 4I_2' - 4I_1' + 6I_2' + 4I_2 \rightarrow 0 = 10I_2' - 4I_1' + 4I_2$$

The above equation is solved by equation (1)

$$0 = 10I_2' - 4\,(-4I_1 + 4I_2 + 12) + 4I_2$$

$$0 = I_2' + 1.6I_1 - 1.2I_2 - 4.8$$

$$I_2' = -1.6I_1 + 1.2I_2 + 4.8 \qquad (2)$$

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ -1.6 & 1.2 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} 12 \\ 4.8 \end{bmatrix},$$

Let $\underline{J'} = \begin{bmatrix} I_1' \\ I_2' \end{bmatrix}, A = \begin{bmatrix} -4 & 4 \\ -1.6 & 1.2 \end{bmatrix}, \underline{J} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, g = \begin{bmatrix} 12 \\ 4.8 \end{bmatrix}$

$$\underline{J'} = A\,J + g \qquad (3)$$

It is a non- homogeneous first order differentiation equation.

Consider a homogeneous differentiation equation is

$$\underline{J'} = A\,J \qquad (4)$$

Let $J_h = x\, e^{\lambda t}$. Then,

$J_h' = \lambda\, x\, e^{\lambda t} \rightarrow (A - \lambda\, I)\boldsymbol{x} = 0 \rightarrow det(A - \lambda\, I) = 0$

$\begin{vmatrix} -4-\lambda & 4 \\ -1.6 & 1.2-\lambda \end{vmatrix} = 0 \rightarrow \lambda_1 = -2 ,\ \lambda_2 = 0.8$

For $\lambda_1 = -2 \rightarrow x_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and

$\lambda_2 = 0.8 \rightarrow x_2 = \begin{bmatrix} 1 \\ 0.8 \end{bmatrix}$

$J_h = c_1 x_1 e^{\lambda_1 t} + c_2 x_2 e^{\lambda_2 t} = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} + c_2 \begin{bmatrix} 1 \\ 0.8 \end{bmatrix} e^{-0.8t}$

Since g is a constant, let $J_p = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ be the particular solution of (3).

Then, $J_p' = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow J_p' = A\,J_p + g$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ -1.6 & 1.2 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} 12 \\ 4.8 \end{bmatrix} \rightarrow a_1 = 3 , a_2 = 0. \therefore\ J_p = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$J = J_h + J_p = wc_1 x_1 e^{-2t} + c_2 x_2 e^{-0.8t} + \boldsymbol{a}$

$I_1 = 2c_1 e^{-2t} + c_2 e^{-0.8t} + 3$

; $I_2 = c_1 e^{-2t} + 0.8c_2 e^{-0.8t}$

$I_1(0) = 0$, $I_2(0) = 0 \rightarrow c_1 = -4 , c_2 = 5$

$I_1 = -8e^{-2t} + 5e^{-0.8t} + 3$ and $I_2 = -4e^{-2t} + 4e^{-0.8t}$

4 CONCLUSION

In my conclusion, matrices can be applied in human's life, i.e., they may be applied not only in various engineering problems, such as in electrical networks, in nets of roads, in production processes, mixing problems etc, but also in economics. All of these were solved in the previous section.

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