

# Performance Improvement of FSO System Through Aperture Averaging of Partially Coherent Beam

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**Abstract:** Performance characteristics of a free space optical communication (FSO) link have been investigated using coherent and partially coherent Gaussian optical beams of wavelengths in the near IR region (785nm to 1550nm). Relatively better performance has been obtained for the partially coherent Gaussian incident light beam compared to the coherent incident beam under moderate and strong atmospheric turbulence conditions. Detailed performance of the FSO link using partially coherent Gaussian beam along with aperture averaging at the receiver have been made. The improvements due to combined effect of partially coherent optical beam and the aperture averaging of the received optical beams are calculated and compared for different values of the aperture widths and wavelengths. Results show that relative improvement in signal to noise ratio and bit error rate performance is more at 1550 nm using partially coherent optical beam.

**Keywords:** FSO, LAN, Scintillation Index, Aperture averaging

## 1. Introduction

Free space optical (FSO) communication systems will play very important role in the future for the design of high bandwidth high data rate short distance communication links as well as for all optical networking. Several FSO communication link design have been proposed using varieties of techniques to get high performance [1-4]. Unfortunately, from such system designs not much stable and high performance system could be designed for the following reasons.

The use of multiple Tran- receiver apertures with equal gain combining (EGB) suggested by Zhu and Kahn and adaptive optical control techniques proposed by [5,6,8] reduces intensity fluctuations but needs complicated electronics circuits. Increasing the size of the receiver aperture offers an effective and simple way to reduce turbulence-induced signal fades but it increases receiver design complicity and background noise level in the received output signal [7]. A compromised solution of how much amount to be increased and in which situation it is useful to increase is still a problem of receiver optimization.

Recently aperture averaging (spatial diversity) [9] effects has been studied to mitigate the influence of scintillation (fading) by using optical coherent beams. In the present paper we investigate the influence of aperture averaging techniques by using partially coherent beam to improve signal to noise ratio and simultaneously to decrease bit error rate for different atmospheric turbulence conditions. First we introduce the basic mathematical model of FSO link then we list the analytical expressions that describe the scintillation index base on kolmogorov turbulence spectrum . In third section Aperture averaging factor “A” is analyzed for both coherent and partially coherent beam. In further section we discuss the performance of FSO link by comparing signal to noise ratio for different optical wavelengths.

## 2. Optical Scintillation Modeling

### 2.1 Propagation Gometry

To study the effect of turbulence on optical beam, consider a lowest order transverse electromagnetic (TEM) Gaussian beam wave (TEM<sub>0,0</sub> wave) . It is assumed that the transmitting aperture located at z=0 and the amplitude distribution is Gaussian with effective beam radius  $W_0$  .

A Gaussian laser beam propagating through the turbulence atmosphere to a receiver along horizontal path distance ‘L’ as shown in Fig.1. The optical field unit amplitude written as [3],

$$U(r,0) = \exp\left(-\frac{r^2}{W_0^2}\right) \text{ at } z=0. \quad (1)$$

where  $r = \sqrt{x^2 + y^2}$

$W_0$  is the initial beam size and x and y are the horizontal and vertical coordinator of the incident beam field from beam center respectively.

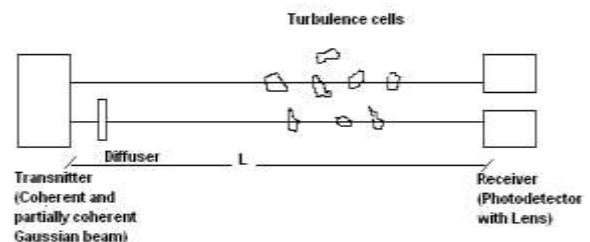


Fig1.Propagation Geometry

The irradiance function of the beam at a distance L from the source can be express as

$$I(x, y, L) = \frac{w_0}{w^2} \exp\left[-\frac{2(x^2 + y^2)}{w^2}\right]$$

(2)

where  $w$  is the average beam size or radius at the receiver

Given by,

$$w = w_0 \left[ 1 + \left( \frac{2L}{kw_0} \right)^2 \right]^{1/2} \left[ 1 + 1.63\sigma^{12/5} \Lambda(L) \right]^{1/2}$$

(3)

where  $k = 2\pi/\lambda$  is wave number,  $\lambda$  is the wavelength of the beam,  $\sigma^2$  is the Rytov variance for plane wave and  $\Lambda(L)$  is the fresnel ratio for vacuum propagation.

## 2.2 Rytov variance :-

The intensity of the optical wave 'I' propagating through turbulent atmosphere is a random variable. The normalized variance of optical wave intensity is defined as,

$$\sigma^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1$$

where the angle bracket denote an ensemble average

$\sigma^2$  indicate the strength of irradiance fluctuations and proportional to Rytov variance, define as

$$\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6}$$

For weak fluctuation, it is less than 1 and for strong fluctuation it is grater than 1.

$C_n^2$  is the refractive index structure constant that characterizes the strength of the index of refraction fluctuations.

Using the kolmogorov spectrum and standard extended Rytov theory [2] the on axis scintillation index for weak turbulence (inner scale  $l=0$ , Outer scale  $L=\infty$ ) is given by.

$$\sigma_{I,w}^2 = 3.86\sigma^2 \left\{ \begin{array}{l} 0.4 \left[ (1 + 2\Theta(L))^2 + 4(\Lambda(L))^2 \right]^{5/12} \\ \cos \left[ \frac{5}{6} \tan^{-1} \left( \frac{1 + 2\Theta(L)}{2\Lambda(L)} \right) \right] \\ - \frac{11}{6} (\Lambda(L))^{5/6} \end{array} \right\}$$

(4)

The fresnel ratio  $\Lambda(L)$  is the beam spread due to diffractions when propagate through air define as ,

$$\Lambda(L) = \frac{\Lambda_0(L)}{\Theta_0^2(L) + \Lambda_0^2(L)}$$

where  $\Lambda_0(L) = \frac{2L}{kw_0^2}$  is the initial fresnel ratio.

$\Lambda(L)$  is associated with the beam curvature parameter  $\Theta(L)$ , which is the phase curvature of the beam as it propagate in vacuum defined as.

$$\Theta(L) = \frac{\Theta_0(L)}{\Theta_0^2(L) + \Lambda_0^2(L)} \text{ where}$$

$$\Theta_0(L) = 1 - \frac{L}{F_0}$$

is the initial phase curvature and for Gaussian beam  $F_0 = \infty$  Hence  $\Theta(L) = 1$

For moderate to strong turbulence scintillation index is

$$\sigma_{I,s}^2 = \exp \left\{ \frac{0.49\sigma_{I,w}^2}{\left[ 1 + 0.56(1 + \Theta) \sigma_{I,w}^{12/5} \right]^{7/6}} + \frac{0.51\sigma_{I,w}^2}{\left( 1 + 0.69\sigma_{I,w}^{12/5} \right)^{5/6}} \right\} - 1$$

(5)

Above equations described on axis scintillation index for point receiver, where diameter  $D \approx 0$ . Now, let us define scintillation index for receiver detector having lens diameter 'D  $\neq 0$ ' [10-11]. For that we assume  $\Omega$  is the normalized receiver aperture and it is independent parameter define as

$$\Omega = \frac{2L}{kW_G^2} \text{ Where } W_G^2 \text{ is the Gaussian lens radius.}$$

Log irradiance due to large scale eddies is given as

$$\sigma_{ln,x}^2(D) = \frac{0.49 \left( \frac{\Omega - \Lambda_1}{\Omega + \Lambda_1} \right) \sigma_{I,w}^2}{\left[ 1 + \frac{0.4 \left( 2 - \bar{\Theta}_1 \right) \left( \frac{\sigma_{I,w}}{\sigma} \right)^{12/7}}{\left( \Omega + \Lambda_1 \right) \left( \frac{1}{3} - \frac{1}{2} \bar{\Theta}_1 + \frac{1}{5} \bar{\Theta}_1^2 \right)^{6/7} + 0.56(1 + \Theta_1) \sigma_{I,w}^{12/5}} \right]^{7/6}}$$

Log irradiance due to Small scale eddies is given as

$$\sigma_{ln,y}^2(D) = \frac{(0.51\sigma_{I,s}^2) \left( 1 + 0.69\sigma_{I,w}^{12/5} \right)^{5/6}}{1 + \left[ 1.20 \left( \frac{\sigma}{\sigma_{I,s}} \right)^{12/5} + 0.83\sigma^{12/5} \right] / (\Omega + \Lambda_1)}$$

Therefore the scintillation index is

$$\sigma_I^2(D) = \exp \left[ \sigma_{ln,x}^2(D) + \sigma_{ln,y}^2(D) \right] - 1$$

### 2.3 Partially coherent beam

If a coherent beam passing through diffuser [13], the phase and amplitude between two random points in an optical beam wanders by significant amount such that the correlation between them partially decreases define as partially coherent beam.

In this section we calculate the scintillation index caused by the combination of diffuser and atmospheric turbulence under weak and moderate to strong conditions.

In the presence of atmospheric effect, we need to take into account some scattering properties caused by the diffuser. Now speckle cells associated with diffuser acts as scattering center with the spatial correlation radius ' $l_c$ ' (cell size) of the diffuser surface produces a separate beam coherence center within the original beam source diameter. Hence, the diffuser acts as an array of independent scattering centers.

The number of scattering centers is given by,

$$N_s = 1 + \frac{2w_0^2}{l_c^2} \quad (6)$$

The effect of diffuser on a optical beam at the receiver is characterized by replacing standard beam parameter  $\Theta_1, \Lambda_1$  by effective beam parameter  $\Theta_{ed}, \Lambda_{ed}$  define in term of  $N_s$  as follows

$$\Lambda_{eff} = \frac{\Lambda_0 N_s}{\Theta_0^2 + \Lambda_0^2 N_s} \quad \text{and}$$

$$\Theta_{eff} = \frac{\Theta_0}{\Theta_0^2 + \Lambda_0^2 N_s}$$

Expressions for partially coherent beam are derived as same as coherent beam equations except the input beam parameters are change due to diffuser located at the transmitter side with different diffuser correlation length  $l_c=0.1, 0.001$ .

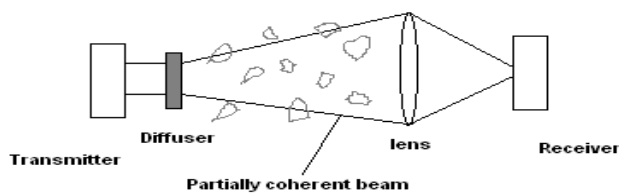


Fig 2. Optical geometry for partially coherent beam

Using the kolmogorov spectrum and standard extended Rytov theory the on axis scintillation index for weak turbulence (inner scale  $l=0$ , Outer scale  $L=\infty$ ) is given by.

$$\sigma_{I,w}^2 = 3.86\sigma^2 \left\{ \begin{array}{l} 0.4 \left[ (1 + 2\Theta_{eff}(L))^2 + 4(\Lambda_{eff}(L))^2 \right]^{5/12} \\ \cos \left[ \frac{5}{6} \tan^{-1} \left( \frac{1 + 2\Theta_{eff}(L)}{2\Lambda_{eff}(L)} \right) \right] \\ - \frac{11}{6} (\Lambda(L))^{5/6} \end{array} \right\}$$

(7)

Where  $\sigma^2$  indicate the strength of irradiance fluctuations and proportional to Rytov variance as

$$\sigma_R^2 = 1.23 C_n^2 k^{7/6} L^{11/6}$$

For weak fluctuation, it is less than 1 and for strong fluctuation it is grater than 1.

$C_n^2$  is the refractive index structure constant that characterizes the strength of the index of refraction fluctuations.

For moderate to strong turbulence scintillation index is

$$\sigma_{I,s}^2 = \exp \left\{ \frac{0.49\sigma_{I,w}^2}{\left[ 1 + 0.56(1 + \Theta_{eff}) \sigma_{I,w}^{12/5} \right]^{7/6}} + \frac{0.51\sigma_{I,w}^2}{\left( 1 + 0.69\sigma_{I,w}^{12/5} \right)^{5/6}} \right\} - 1$$

(8)

Now, let us define scintillation index for receiver detector having lens diameter 'D'. For that we assume  $\Omega$  is the normalized receiver aperture and it is independent parameter define as

$$\Omega = \frac{2L}{kW_G^2}$$

where  $W_G^2$  is the Gaussian lens radius.

Log irradiance due to large and small scale eddies is given as

$$\sigma_{P \ln, x}^2(D) = \frac{0.49 \left( \frac{\Omega - \Lambda_{eff}}{\Omega + \Lambda_{eff}} \right) \sigma_{PI,w}^2}{\left[ 1 + \frac{0.4(2 - \bar{\Theta}_{eff}) \left( \frac{\sigma_{PI,w}}{\sigma} \right)^{12/7}}{\left( \Omega + \Lambda_{eff} \right) \left( \frac{1}{3} - \frac{1}{2} \bar{\Theta}_{eff} + \frac{1}{5} \bar{\Theta}_{eff}^2 \right)^{6/7} + 0.56(1 + \Theta_1) \sigma_{I,w}^{12/5}} \right]^{7/6}}$$

$$\sigma_{P \ln, y}^2(D) = \frac{(0.51\sigma_{PI,s}^2) / \left( 1 + 0.69\sigma_{PI,w}^{12/5} \right)^{5/6}}{1 + \left[ 1.20 \left( \frac{\sigma}{\sigma_{PI,s}} \right)^{12/5} + 0.83\sigma^{12/5} \right] / (\Omega + \Lambda_{eff1})}$$

$$\sigma_{P,I}^2(D) = \exp[\sigma_{P_{ln,x}}^2(D) + \sigma_{P_{ln,y}}^2(D)] - 1. \quad (9)$$

### 3. Aperture Averaging Factor

After propagating through atmospheric turbulence in the channel the quality of the beam at the receiver deteriorates and begins to break up into random regions of high and low intensity. Increasing the size of the receiver aperture relative to the size of these regions of low intensity averages the signal fluctuations and thus decreasing the signal fading, this phenomenon is known as “aperture averaging”.

The aperture averaging factor ‘A’ is defined [12,15-16] by the normalized variance of power fluctuations of the incident optical field on collecting lens. It is the ratio of the irradiance flux variance obtained by a finite-size collecting lens having diameter D to that obtained by a point receiver or on axis flux variance.

From equations (5) and (6) we can define Aperture averaging factor [8] for coherent beam as

$$A = \sigma_I^2(D) / \sigma_{I,s}^2(0). \quad (10)$$

Where  $\sigma_I^2(D)$  &  $\sigma_{I,s}^2$  are the scintillation index for receiver lens of diameter D and a “point receiver” (D=0) respectively.

Under moderate to strong turbulence conditions, only eddies of size smaller than coherence radius  $\rho_0$  or larger than the scattering disk  $L/k\rho_0$  contribute effectively [9] [10] to the atmospheric turbulence. To take into account this dependence of the turbulence on Coherent and partially coherent beams, it is proposed to consider an Aperture averaging factor ‘A’ to mitigate the effects scintillation by considering the three special cases of weak, moderate and strong turbulence with different wavelengths. The effects of aperture averaging on the received signal is given by

$$\sigma_{I,s}^2 = A * \sigma_{I,s}^2 \quad (11)$$

Where  $\sigma_{I,s}^2$  aperture is averaged scintillation index.

### 4. Signal to noise ratio and Bit error rate

Signal to noise ratio (SNR) depends on averaged receive power, the scintillation over the aperture and the receiver noise. It also depends strongly on the decision level setting in the receiver for bit 1 and 0. Now, aperture averaging affects both the received power and its scintillation. The following analysis follows the effect of aperture averaging on signal to noise ratio (SNR) and BER in turn the overall performance of the FSO link.

First, we define the output SNR in the absence of optical turbulence by the ratio of the detector signal current  $i_s$  to the root-mean-square (rms) noise current-  $\sigma_N$ , which yields

$$SNR_0 = \frac{i_s}{\sigma_N} = \sqrt{\frac{\eta P_s}{2h\nu B}}, \quad i_s = \frac{\eta e P_s}{h\nu}$$

where  $i_s$  - is signal current,  $P_s$  is the signal power in watts,  $B$  - filter bandwidth,  $\eta$  is the detector quantum efficiency in electrons/photon,  $e$  is electric charge in coulombs,  $h$  is the Planck’s constant ( $h = 6.63 * 10^{-34}$  joule-second) and  $\nu$  is optical frequency in hertz.

We use the most basic form of pulse modulation is on –off keying (OOK). Each bit symbol is transmitted by pulsing the source either on or off during each bit interval. Because of random noise, a transmitted 0 may be mistaken for a 1, which is denoted by  $\Pr(1/0)$ , and 1 may be mistaken for a 0, denoted by  $\Pr(0/1)$ . Assuming each symbol is equally likely to be sent, the BER is given by.

$$\Pr(E) = \frac{1}{2} \Pr(1/0) + \frac{1}{2} \Pr(0/1) = \frac{1}{2} \operatorname{erfc}\left(\frac{SNR_0}{2\sqrt{2}}\right)$$

In the presence of atmospheric turbulence, the received signal exhibits additional power losses (refraction, diffraction) and random irradiance fluctuations.

Therefore SNR becomes

$$\langle SNR \rangle = \frac{SNR_0}{\sqrt{\left(\frac{P_{SO}}{\langle P_S \rangle}\right) + \sigma_I^2(D) SNR_0^2}} \quad (12)$$

where  $P_{SO}$  is the signal power in the absence of atmospheric effects and  $\sigma_I^2(D)$  is the irradiance flux variance on the photo detector.  $\langle \rangle$  represent mean.

The power ratio  $\frac{P_{SO}}{\langle P_S \rangle}$  provides a measure of SNR

deterioration caused by atmospheric induced beam spreading given by

$$\frac{P_{SO}}{\langle P_S \rangle} = 1 + 1.63 \sigma_R^{12} \Lambda_1$$

In the presence of optical turbulence, the probability of error is given by

$$\Pr(E) = \langle BER \rangle = \frac{1}{2} \int_0^{\infty} p_I(u) \operatorname{erfc} \left( \frac{\langle SNR \rangle u}{2\sqrt{2}} \right) du$$

Where  $p_I(u)$  is a gamma-gamma distribution with unit mean

$$p_I(u) = \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} u^{\frac{(\alpha+\beta)}{2}-1} K_{\alpha-\beta} \left( 2\sqrt{\alpha\beta}u \right) \text{ for } u > 0$$

When aperture averaging effects are consider, parameters  $\alpha$  and  $\beta$  for the gamma-gamma PDF are define as

$$\alpha = \frac{1}{\exp[\sigma_{\ln X}^2(D)] - 1} ;$$

$$\beta = \frac{1}{\exp[\sigma_{\ln Y}^2(D)] - 1}$$

### 5. Numerical Result

In this section we compared SNR of two FSO system which uses same input parameter except beam type is coherent and partially coherent respectively.

All simulation was realized in the MATLAB environment with beam parameters considered as follows.

Wavelength

$$\lambda = 1550nm, 1310nm, 980nm \text{ \& } 780nm.$$

Distance  $L = 2000m$ , Refractive Index structure parameter

$C_n = 10^{-12}, 10^{-14} \text{ \& } 10^{-16} m^{-\frac{2}{3}}$  for weak, moderate and strong turbulence conditions respectively [14]. Beam width  $W$  and diameter  $D$  of receiver aperture for both coherent and partially coherent beam is chosen as 0.025m and 0.08m. First, we compare scintillation index for both the beam using different wavelengths shown in Fig.3 and Fig.4. Same case using aperture averaging techniques for both beams with different wavelength from 1550 nm to 785nm with refractive index structure parameter for weak turbulence  $C_n = 10^{-16}$  shown in following fig.5 and fig.6. In last case we compare SNR for both types using same conditions. It is observe that using partially coherent beam with aperture averaging SNR is improved compared to coherent beam.

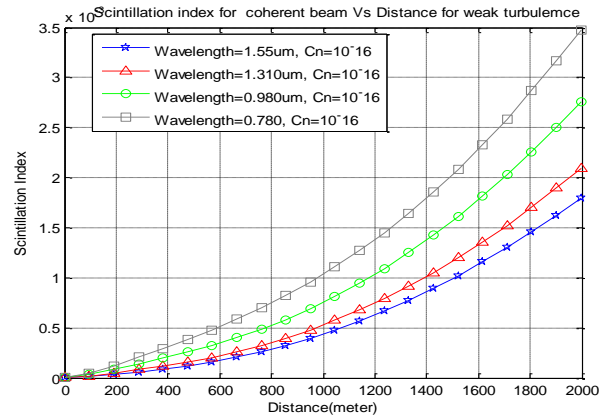


Fig.3 Scintillation index for coherent beam Vs Distance for weak turbulence

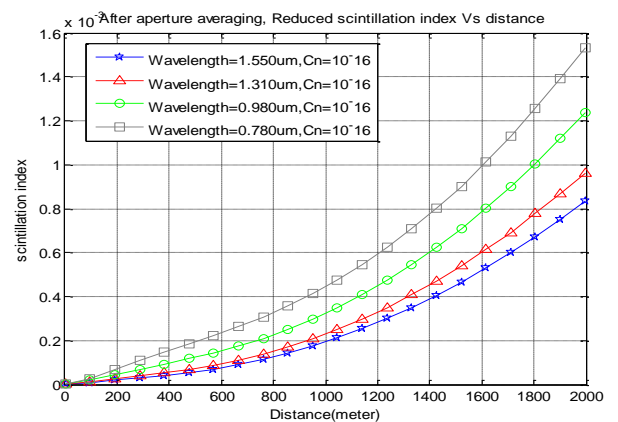


Fig4. After aperture averaging, Reduced scintillation index Vs distance

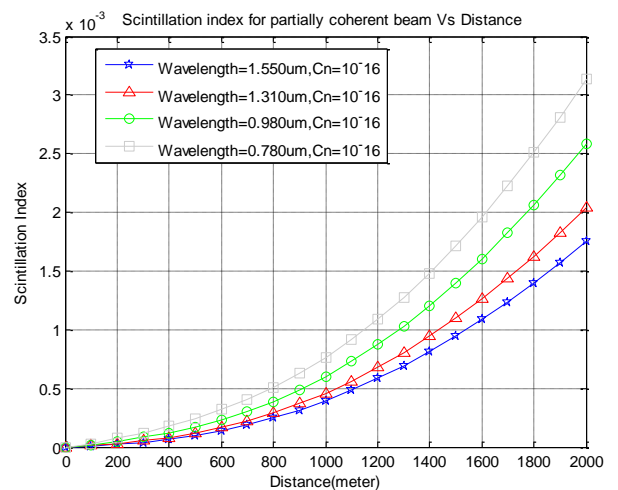


Fig.5 Scintillation index for partially coherent beam Vs Distance for weak turbulence

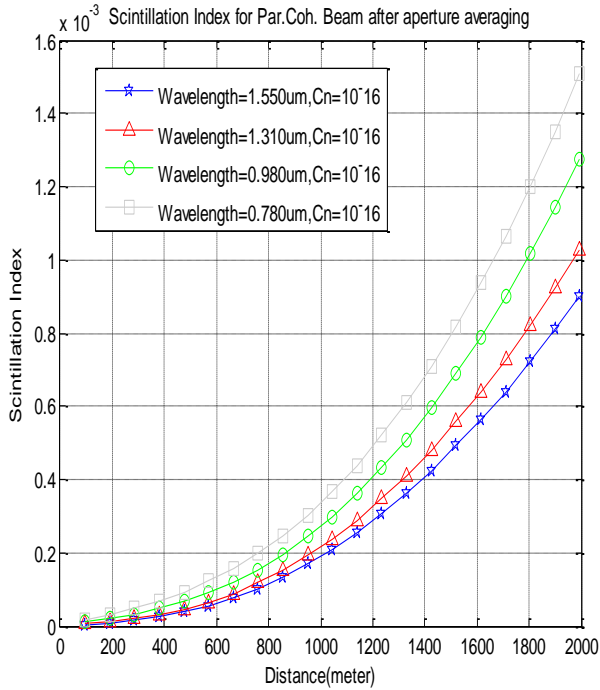


Fig 6. Scintillation Index for Partially Coh. Beam after aperture averaging

Now let us compare signal to noise ratio for weak turbulence condition ( $C_n = 10^{-6}$ ) using coherent and partially coherent beam using same initial conditions.

We assume the ratio  $\frac{P_{so}}{P_s} = 1$ , and from above figure for 1.5

km distance the value of aperture averaged scintillation index for both coherent and partially coherent beam is 0.6 and 0.5.(for 1550nm wavelength) Therefore from equation (12) we have

$$\langle SNR \rangle = \frac{SNR_o}{\sqrt{1 + \sigma_i^2(D)SNR_o^2}} \quad (13)$$

For different values of SNR<sub>o</sub>, mean SNR is plotted in Fig 7 and Fig8. which clearly shows 30% improvement in signal to noise ratio due aperture averaging techniques for 1550nm wavelength.

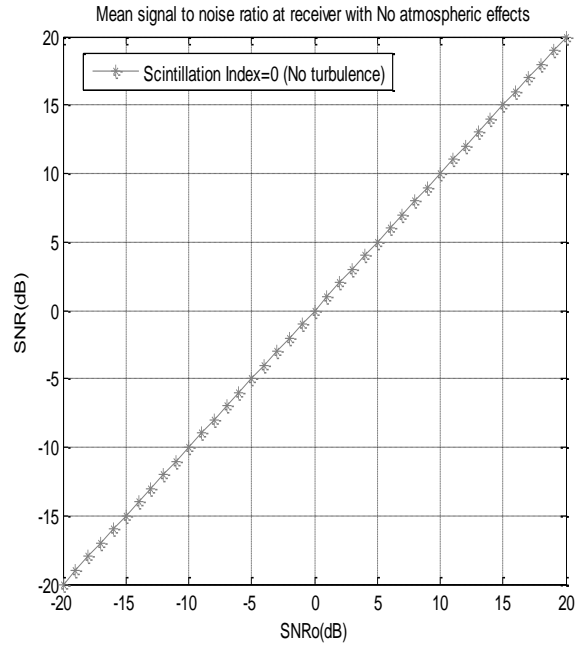


Fig.7 MeanSNR Vs SNR<sub>o</sub> for Zero turbulence

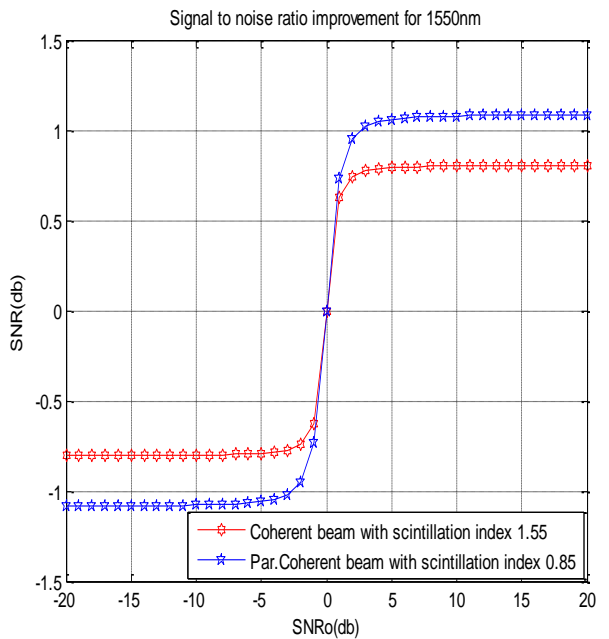


Fig.8 Improvement in SNR using aperture averaging of Partially coherent beam

## 6. Conclusion

The atmospheric turbulence effect on the performance of a 2 km long FSO link is studied under moderate to strong atmospheric turbulence conditions. The performance of the system was studied at the 780nm, 980nm, 1310 nm and 1550nm wavelengths by analyzing SNR for different atmospheric turbulence conditions. Results are verified using MATLAB simulation. It is observed that, scintillation index variations is more pronounced for 785nm coherent Gaussian beam compared to partially coherent beam. As the wavelength is increases more and more averaging is take place at the receiver side i.e. improved SNR is obtained at 1550nm wavelength for partially coherent beam.

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