# Application of Laplace Transform for RLC Circuit

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**Abstract**: In this paper, Laplace transform is discussed and electric circuit problem as second order nonhomogeneous linear ordinary differential equation with constant coefficients is formulated. Then, this problem is solved by using Laplace transform method and analytical method. And then, ELC circuit acting over a time-interval will be solved by applying only Laplace transform method.

**Keywords**: Laplace transform. Ordinary differential equation. Electric circuit. Kirchhoff's Voltage Law. Time-shifting.

## 1. INTRODUCTION

The Laplace transform is an integral transform in mathematics. The transform has many applications in science and engineering such as first order ODE modeling (RL & RC)circuits with no AC source and with a DC source, second order ODE modeling (series & parallel RLC) circuits with no DC source and with AC source, and so on. Laplace transform of unit step function is suitable for solving ODEs with complicated right sides of considerable engineering interest such as single waves, inputs (driving forces) act for some time only. Laplace transforms are usually restricted to functions of $t$ with $t \geq 0$. Laplace transformation from the time domain to the frequency domain transforms second order ordinary differential equations into algebraic equations. The required solutions are obtained by applying definition and some properties of Laplace transform as follows.

### 1.1 Definition of Laplace transform:

Let $y(t)$ be a function of $t$ specified for $t > 0$, then the integral $\int_0^\infty e^{-st} y(t)dt$ is called Laplace transform of $y(t)$ and is denoted by $L\{y(t)\}$ or Y(s).

i.e. $L\{y(t)\} = \int_0^\infty e^{-st} y(t)dt = Y(s)$

### 1.2 Properties of Laplace transform:

Linearity property:

If $f(t) and\ g(t)$ are any functions of t, a and b are any constants, then

$$L\{af(t) + bg(t)\} = aL\{f(t)\} + bL\{g(t)\}.$$

Laplace transform of derivatives:

If $L\{y(t)\} = Y(s)$, then

$$L\{y^{(n)}(t)\} = s^n Y(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \cdots - y^{(n-1)}(0).$$

Inverse Laplace transform:

If $L\{y(t)\} = Y(s)$, then $y(t) = L^{-1}\{Y(s)\}$ is called inverse Laplace transform of Y(s).

S-shifting property:

If $L\{y(t)\} = Y(s)$, then $L\{e^{at}y(t)\} = Y(s-a)$.

Laplace transform of unit step function:

The unit step function is a typical engineering function made to measure for engineering applications, which often involve functions are either "off" or "on".

$$u(t-a) = \begin{cases} 0 \ if \ t < a \\ 1 \ if \ t > a \end{cases}$$

is also called Heaviside function.

T-shifting Property:

If $L\{y(t)\} = Y(s)$, then $L\{y(t-a)u(t-a)\} = Y(s)e^{-as}$

which is often modeled in a RLC circuit by a voltage source in series with a switch.

## 2. APPLICATION OF LAPLACE TRANSFORM ON ODES

This section, the definition of ordinary differential equation and the application of Laplace transform on second order linear ODE are described.

An ordinary differential equation (ODE) is a differential equation containing one or more functions of one independent variable and the derivatives of those functions.

The Laplace transform is a useful method in solving linear ODE with constant coefficients.

Consider second order nonhomogeneous initial value problem

$$y'' + ay' + by = r(t), \ y(t_0) = k_0, y'(t_0) = k_1 \quad (1)$$

If Laplace transform on both sides of (1) is taken, the Laplace transform of derivatives and initial conditions are used, then algebraic equation is got. And then, the required solution is obtained by applying the inverse Laplace transform and s-shifting property.

Example: $y'' + 4y' + 20y = 120, \ y(0) = 0, y'(0) = 0.$

Solution: Taking Laplace transform on both sides

$$L[y''] + 4L[y'] + 20L[y] = L[120]$$

Using Laplace transform of derivatives and initial condition

$$(s^2+4s+20)Y = \frac{120}{s}$$

$$\therefore \quad Y(s) = \frac{120}{s(s^2+4s+20)}$$

By using partial fraction method, inverse Laplace transform, and s-shifting property

$$y(t) = 6t - 6e^{-2t}\cos 4t - 3e^{-2t}\sin 4t.$$

## 3. MODELING RLC CIRCUIT

This section, the definition of electric circuit, Kirchhoff's Voltage Law and modeling to RLC circuit according to KVL are presented.

An electric circuit is a path in which electrons from a voltage or current source flow. The point where those electrons enter an electric circuit is called the source of electrons high-voltage direct current transmission uses big converters.

Kirchhoff's Voltage Law states that the sum of all voltages around a closed loop in any circuit must be equal to zero. This is a consequence of charge conservation and also conservation of energy.

Consider the circuits are basic building blocks of such networks. They contain three kinds of components, a resistor of resistance R Ω(ohms), an inductor of inductance L H(henrys) and a capacitor of capacitance C F(farads) are wired in series circuit, the same current flows through all components of the circuit, and connected to a generator or an electromotive force E(t) V(volts), sinusoidal as in following figure

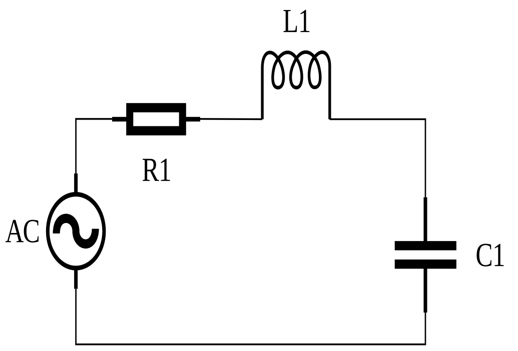


Figure1. RLC-circuit

The circuit is a closed loop, and the impressed voltage E(t) equals the sum of the voltage drops across the three elements R, L,C of the loop.

According to Kirchhoff's Voltage Law, the above figure for an RLC-circuit with electromotive force

$E(t) = E_0 \sin \omega t (E_0\, constant)$ as a model

$$E_L + E_R + E_C = E(t) \quad (2)$$

$$Li' + Ri + \frac{1}{C}\int i\, dt = E(t) = E_0 \sin \omega t$$

or $q'' + Rq' + \frac{1}{C}q = E_0 \sin \omega t$ (3)

here q is the charge on the capacitor, i is the current in the circuit : $i(t) = \frac{dq}{dt}$ $or$ $q(t) = \int i(t)dt$

and differentiate (3)

$$Li'' + Ri' + \frac{1}{C}i = E_0 \cos \omega t. \quad (4)$$

This equation is a modeling RLC circuit as a second-order non-homogeneous linear ODE with constant coefficients.

## 4. APPLICATION TO RLC- CIRCUIT

Many science and technical problems are built as mathematical model in various fields. These models are solved by applying kinds of mathematical methods. Among them, now present modeling RLC circuit as second order non homogeneous linear ODE with constant coefficients by applying the analytical method and Laplace transform method. And then, solve RLC circuit problem given time interval by applying Laplace transform of time shifting property.

### 4.1 Analytical and Laplace transform methods application to RLC-circuit problem

A circuit has in series an electromotive force of 600 V, a resistor of 24 Ω, an inductor of 4 H, and a capacitor of $10^{-2}$ farads. If the initial current and the initial charge on the capacitor are both zero, Find the charge and the current at time t>0.

According to Kirchhoff's Voltage Law

$$E_L + E_R + E_C = E(t)$$

$$Lq'' + Rq' + \frac{1}{C}q = 600$$

$$q'' + 6q' + 25q = 150 \quad (5)$$

is second-order non homogeneous linear ODE.

Applying Analytical method

The corresponding homogeneous linear ODE of (5) is

$$q'' + 6q' + 25q = 0 \quad (6)$$

the corresponding characteristic equation of (6) is

$$\lambda^2 + 6\lambda + 25 = 0 \text{ and } \lambda = -3 \pm 4i$$

the general solution of (6) is

$$q_h(t) = e^{-3t}(A\cos 4t + B\sin 4t)$$

using the method of undetermined coefficients, $q_p(t) = 6$

then, the general solution of (5) is

$$q(t) = e^{-3t}(A\cos 4t + B\sin 4t) + 6 \quad (7)$$

differentiating (7) and using initial condition, the charge:

$$q(t) = 6 - 6e^{-3t}\cos 4t - 4.5e^{-3t}\sin 4t$$

and the current: $i(t) = 37.5e^{-3t}\sin 4t.$

Applying Laplace transform method

Taking Laplace transform on both sides of (5)

$$L\{q''\} + 6L\{q'\} + 25L\{q\} = L\{150\}$$

using Laplace transform of derivative and initial condition

$$Q(s) = \frac{150}{s(s^2+6s+25)}$$

by partial fraction method,

$$Q(s) = \frac{6}{s} - \frac{6(s+3)+18}{(s+3)^2+4^2}$$

applying inverse transform and s-shifting property,

the charge:

$$q(t) = 6 - 6e^{-3t}\cos 4t - 4.5e^{-3t}\sin 4t$$

and the current: $i(t) = 37.5e^{-3t}\sin 4t.$

### 4.2 Output of an RLC-circuit to a sinusoidal input acting over a time interval

An inductance of 0.4 henry, a resistor of 12 ohms and a capacitor of 0,0125 farad are connected in series with an electromotive force of 220 sin 10t volts. At t = 0, the charge on the capacitor and current in the circuit is zero. Find the current where E(t) is sinusoidal, acting for a short time interval

$$E(t) = \begin{cases} 220\sin 10t & if\ 0 < t < 2\pi \\ 0 & if\ \ t > 2\pi \end{cases}$$

Modeling by KVL,

$$Li'' + Ri' + \frac{1}{C}i = E', \qquad i(0) = 0, i'(0) = 0$$

$$i'' + 30i' + 200i = 550 \times 10\cos 10t.$$

The above equation is second-order nonhomogeneous linear ODE.

Applying Laplace transform of derivative and time shifting property

$$[s^2I(s) - si(0) - i'(0)] + 30[sI(s) - i(0)] + 200I(s)$$

$$= 5500\left[\frac{s}{s^2+10^2}(1 - e^{-2\pi s})\right]$$

using initial conditions and partial fraction method,

$$I(s) = \frac{22}{s+20} - \frac{27.5}{s+10} + \frac{5.5s}{s^2+10^2} + \frac{165}{s^2+10^2}(1 - e^{-2\pi s})$$

Applying the inverse Laplace transform on both sides and t-shifting property, the current is

$$i(t) = 22\left(e^{-20t} - e^{-20(t-2\pi)}\right) - 27.5(\left(e^{-10t} - e^{-10(t-2\pi)}\right).$$

## 5. CONCLUSIONS

Through this paper, we present the application of Laplace transform and RLC-circuit is modeled as second order nonhomogeneous linear ODE. When RLC-circuit problem is solved by applying the two methods, the charge and the current of this problem are the same in subsection 4.1. But, RLC-circuit acting over time interval can be solved by applying only Laplace transform method in subsection 4.2. Therefore, linear ordinary differential equations with constant coefficients can be easily solved by the Laplace Transform method without finding the general solution, particular solution and the arbitrary constants as analytical method. Thus, Laplace transform method is more effective tool to solve complex problems than the analytical method in various fields.

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