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Workspace Analysis of Two-link Planar Manipulator

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Abstract: A two link revolute robotic arm is optimized for maximization of work space area covered by its end effector. A mathematical model for optimization is built considering singularities which influence the variation of design variables. Condition number which is the measure of output value (End effector position) for a small change in input value (joint angles) is modelled as the constraint. Joint angle between link2 and link1 and link lengths are considered as design variables. The mathematical model is initially optimized using Semi-infinite Programming technique. Genetic Algorithm using Roulette wheel selection is employed on the nonlinear optimization model for obtaining global optimum value for the objective function. The maximum value of objective function obtained from Genetic Algorithm is found to be considerably higher than the value obtained from Semi-infinite Programming method.

Keywords: Kinematics and Dynamics, Denavit-Hartenberg (DH), 2DOF manipulator, MatLab

1. INTRODUCTION

The objective of designers is to maximize the area covered by the robotic end effectors in their work space. However, their movements are restricted due to kinematic singularities. It is popularly known fact that the Jacobian matrix relates linear velocities of links or end effectors to their joint velocities. At singular positions, the determinant of jacobian becomes zero resulting in infinite joint velocities. Therefore, within these restrictions (singularities), it is area of interest of many researchers to find the global optimum which gives maximum reach for the end effector. A.Morecki et.al (1984) discussed the effect of link length ratios on the distance travelled by end effector.

Position analysis is an essential step in the design, analysis and control of robots. In this article, a basic introduction to the position analysis of serial manipulators is given. This topic is invariably covered in all the textbooks on this subject. Therefore, instead of repeating the standard details of forward kinematics, such as, the designation of the reference frames, determination of the Denavit-Hartenberg (DH) parameters, multiplication of the 4×4 transformation matrices to get the end-effector position and orientation etc., more emphasis is laid on the inverse problem, which is relatively more complicated in such manipulators.

A vertical revolute configuration, a 2-R robot with two degrees of freedom is generally well-suited for small parts insertion tasks for assembly lines like electronic components insertion. Although the final aim is real robotics, it is often very useful to perform simulations prior to investigations with real robots. This is because simulations are easier to setup, less expensive, faster and more convenient to use [2].

In this paper we examine in detail the description of the workspace of a two degree of freedom planar robot manipulator. We use the technique based on polynomial discriminants described in to characterize the workspace from an algebraic point of view. With the algebraic representation, we can easily investigate the effects of kinematic parameters on the workspace. Most importantly, the algebraic representation of a robot workspace facilitates simple solutions to many problems in the analysis of robot manipulators.

2. MATHEMATICAL MODEL

A 2R planar serial robot is considered as the mathematical modeling for kinematically redundant manipulator. Where l_1 and l_2 are the lengths of the link 1 and 2, and θ_1 and θ_2 are joint variables. P_x and p_y are the position of robot end-effector. The workspace of this planar robot isthen all P_x and p_y pairs that the robot is able to reach. Both joints can rotate 360°, the workspace is simply a circle of radius $l_1 + l_2$. When limits exist, the workspace becomes more complex.

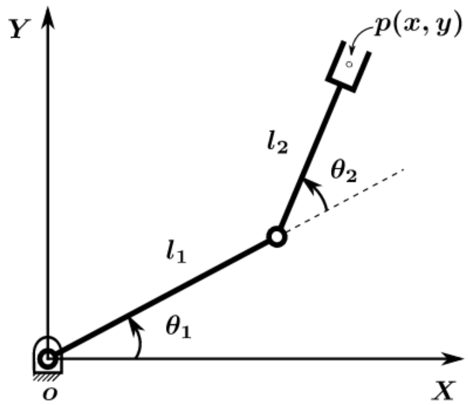

Figure. 1 Two link planar manipulator

Figure. 1 shows a schematic representation of two link planar manipulator with link lengths and link angles.

2.1 Kinematics of 2R planar manipulator

From forward kinematics the following relations connecting end effector positions with joint angles and link lengths are obtained.

$$p_x = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \quad (1)$$

$$p_y = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \quad (2)$$

The first kinematic equation in terms of a single joint variable, θ_1, is obtained by adding the square of equations (1) and (2),

$$\begin{aligned} g_1(\theta_1) &= p_x^2 + p_y^2 - 2l_1 p_x \cos\theta_1 \\ &-2l_1 p_y \sin\theta_1 + l_1^2 - l_2^2 = 0 \end{aligned} \quad (3)$$

To obtain the other independent kinematic equation in terms of θ_2, rewrite equations (1) and (2) as

$$p_x = l_1 \cos\theta_1 + l_2 \cos\theta_1 \cos\theta_2 - l_2 \sin\theta_1 \sin\theta_2 \quad (4)$$

$$p_y = l_1 \sin\theta_1 + l_2 \sin\theta_1 \cos\theta_2 - l_2 \cos\theta_1 \sin\theta_2 \quad (5)$$

Multiplying (4) by $\sin\theta_1$ and (5) by $\cos\theta_1$ and subtracting, equation (6) is obtained.

$$\cos\theta_1 p_y + \sin\theta_1 p_y = l_2 \sin\theta_2 \quad (6)$$

Now equations (3) and (6) are linear equations in $\sin\theta_1$ and $\cos\theta_1$ and can be used to eliminate $\sin\theta_1$ and $\cos\theta_1$ to obtain an expression that relate p_x and p_y and link lengths to θ_2. Specifically,

$$p_x^2 + p_y^2 = l_2^2 \sin^2\theta_2 + \left[\frac{p_x^2 + p_y^2 + l_1^2 - l_2^2}{2l_1}\right]^2 \quad (7)$$

which can be simplified to

$$g_2(\theta_2) = p_x^2 + p_y^2 - l_1^2 - l_2^2 - 2l_1 l_2 \cos\theta_2 = 0 \quad (8)$$

Equations (3) and (8) can be verified graphically using in Fig. 1 and they are the two independent kinematic equations on which workspace description is based. Since they are in term of revolute joint variables, substitution

$$x = \tan\frac{\theta_i}{2}$$

or

$$\sin\theta_i = \frac{2x}{1+x^2} \quad \text{and} \quad \cos\theta_i = \frac{1-x^2}{1+x^2} \quad (9)$$

is necessary to remove the periodicity. Then the set of workspace boundary surfaces as required by equation (3), with substitution equation (9), are given by the roots of the characteristic polynomial

$$p_x^2 + p_y^2 - 2l_1\left(p_x \frac{1-x^2}{1+x^2} + p_y \frac{2x}{1+x^2}\right) = l_2^2 - l_1^2$$

or

$$\begin{aligned} &x^2(p_x^2 + p_y^2 + l_1^2 - l_2^2 + 2l_1 p_x) + x(-4l_1 p_y) \\ &\quad + \left[p_x^2 + p_y^2 - 2l_1 p_x + (l_1^2 - l_2^2)\right] = 0 \end{aligned} \quad (10)$$

(10) is a polynomial in x and has the form $C_2x^2+C_1x+C_0$. Its real roots cease to exist when $C_1^2-4C_0C_2$ is zero, or

$$p_x^2 + p_y^2 = (l_1 + l_2)^2 \quad \text{and} \quad p_x^2 + p_y^2 = (l_1 - l_2)^2 \quad (11)$$

Turning now to the second kinematic equation (8), substitution (9) into (8) results in the characteristic polynomial

$$p_x^2 + p_y^2 - l_2^2 - l_1^2 - 2l_1 l_2 \frac{1-x^2}{1+x^2} = 0$$

or

$$\begin{aligned} &x^2(p_x^2 + p_y^2 - l_1^2 - l_2^2 \pm 2l_1 l_2) \\ &+(p_x^2 + p_y^2 - l_1^2 - l_2^2 \pm 2l_1 l_2) = 0 \end{aligned} \quad (12)$$

The boundary surfaces defined by θ_2 coincide with those defined by θ_1. In addition, the boundary surfaces in both cases are two concentric circles with centers at the origin of the x-y plane and with radius of $|l_1+l_2|$ and $|l_1-l_2|$, respectively. The second circle degenerates to a point at the origin when the links have the same length. Therefore, the workspace description derived by the algorithm in verifies that which we commonly take for granted.

Additional boundary surfaces are introduced when joint limits are considered. These surfaces are defined by substituting the limit values into the corresponding kinematic equation, equation (3) for joint one and equation (8) for joint two. For the planar robot under investigation, assume

$$\theta_{1\min} \le \theta_1 \le \theta_{1\max} \quad \text{and} \quad \theta_{2\min} \le \theta_2 \le \theta_{2\max}$$

To derive the additional boundary surface due to joint one limits, rewrite the kinematic equation (3) as

$$(p_x - l_1 \cos\theta_1)^2 + (p_y - l_1 \sin\theta_1)^2 = l_2^2 \quad (13)$$

With either joint one limit substituted into (13), we obtain another circle centered at ($l_1\cos\theta_{1m}$, $l_1\sin\theta_{1m}$), where θ_{1m} is either the minimum or maximum joint limit.
Similarly if one rewrites equation (8) as

$$p_x^2 + p_y^2 = l_1^2 + l_2^2 + 2l_1 l_2 \cos\theta_2 \quad (14)$$

Equation (14) produces tow more circle with centers at the origin and with radius l where

$$l^2 = l_1^2 + l_2^2 + 2l_1 l_2 \cos\theta_{2m}$$

Note that it can be easily shown that

$$(l_1 - l_2)^2 \le l^2 \le (l_1 + l_2)^2$$

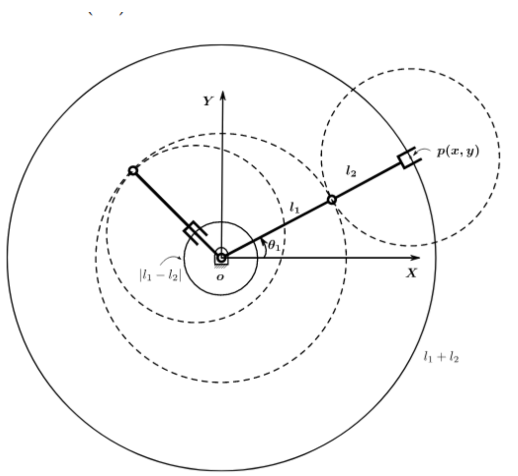

Figure. 2 Singularities and workspace of the 2R planar manipulator

The circles delimit the set of points in the plane that the manipulator can reach. In other words, these circles define the boundaries of the workspace (or more precisely, the reachable workspace) of the manipulator.

3. EFFECTS OF KINEMATIC PARAMETERS ON WORKSPACE

Each workspace boundary surface reveals little of how it is relate to other boundary surface. The complete workspace is not simply the intersection of workspaces obtained by considering link and joint constraints individually. For the planar robot, the nature of the workspace depends on six parameters, two link lengths and four joint limits. We take the approach to first consider the simplest situation where joint limits do not exit, and gradually introduce joint limits as additional constraints on the workspace geometry.

Without Joint Limits, the workspace is constrained only by the link lengths. As stated previously, the geometry of the workspace in this case is a ring defined by two concentric circles of radious $|l_1+l_2|$ and $|l_1-l_2|$ with centers at the origin. The inner circle may possible degenerate to a singular point at the origin when the two links are of the same length.

3.1 Consideration of Joint 1 Limits

Two additional boundary surfaces result from limits of joint 1 and they are defined by (21) when the two limits are substituted into it. Part of the two circles will form the boundary of the final workspace. The workspace contribution can be analyzed for three different case, $l_1 > l_2$, $l_1 = l_2$ and $l_1 < l_2$.

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3.2 Consideration of Joint 2 Limits

When joint 1 can rotate a complete revolution while θ_2 is restricted to (θ_{2min}, θ_{2max}), according to two additional curves, circles in this case. It can possibly form new workspace boundaries, and radii of the two circles are defined by

$$r_{21} = \sqrt{l_1^2 + l_2^2 + 2l_1 l_2 \cos_{2\min}}$$

$$r_{21} = \sqrt{l_1^2 + l_2^2 + 2l_1 l_2 \cos_{2\max}}$$

4. RESULTS

Three workspace results presentations are shown in Figure. 2, Figure. 3 and Figure. 4. The workspaces are plotted in MatLab Software using forward kinematic equation for two link planar manipulator.

Case I: $l_1 < l_2$

In this case link 2 is longer than link 1 and the workspace typically has the shape of an apple with its core taken out. A workspace is shown in Fig. 3. Where the set of kinematic parameters are l_1 = 0.23 m, l_2 = 0.17 m and 0° <θ_1 <180°.

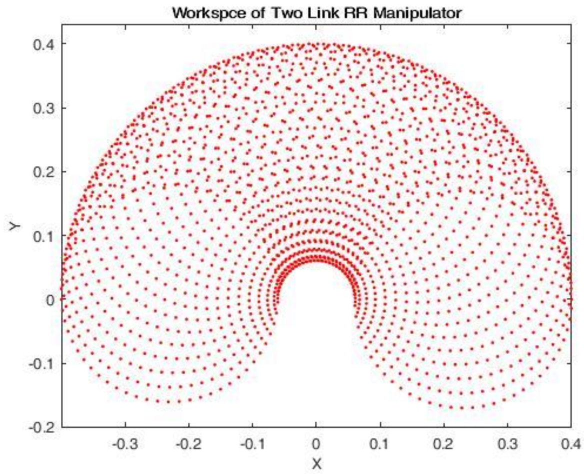

Figure. 3 Workspace with joint 1 limit: Case I

Case II: $l_1 = l_2$

When the two links are of the same length, the workspace looks either like an apple without a core if the range of the joint 1 is greater than or equal to 180°, or like an apple with an oval-shaped core. A workspace is shown in Fig. 4. Where the set of kinematic parameters are l_1 = 0.20 m, l_2 = 0.20 m and 0° <θ_1 <180°.

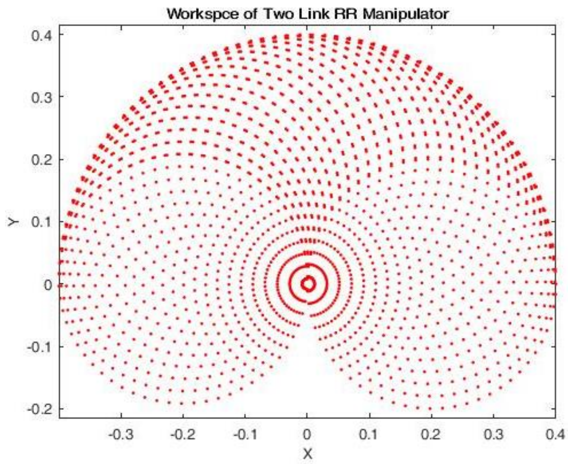

Figure. 4 Workspace with joint 1 limit: Case II

Case III: $l_1 > l_2$

In this case, the workspace looks either like an apple with a core of a sharp tip pointing to the apple top. A workspace is shown in Figure. 4. Where the set of kinematic parameters are l_1 = 0.17 m, l_2 = 0.23 m and 0° <θ_1 <180°.

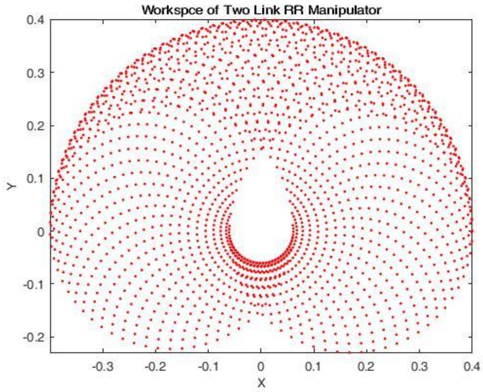

Figure.5 Workspace with joint 1 limit: Case III

5. CONCLUSION

In this paper, the complete mathematical formulation for forward kinematics of two link planar robot manipulator having two degree of freedom is derived. An analytical formulation for determining the robot workspace is presented. This formulation is used to develop the MatLab tool box. The workspace constraint function was formulated in terms of generalized coordinates including, inequality constraints imposed on each joint. For the computational analysis of mathematical formulation of complete forward kinematics of the system MATLAB code are developed in the form of several M-files. The simulated results are also plotted.

6. ACKNOWLEDGMENTS

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